Faculty of Education
Mathematics Department

ON ORDERED RANDOM VARIABLES AND ITS CONCOMITANTS FOR SOME CONTINUOUS DISTRIBUTIONS

A Thesis

Submitted in Partial Fulfillment of the Requirements of the Doctor of Philosophy Degree in Teacher's Preparation in Science

(Statistics)

Submitted to:

The Department of Mathematics, Faculty of Education, Ain Shams University

By

Mohamed Said Mohamed Mahmoud

Ass. Lecturer at,
Mathematics Department, Faculty of Education, Ain Shams University

Supervised by

Prof. Dr. Mostafa M. Mohie El-Din
Professor of Mathematical Statistics
Faculty of Science
Al-Azhar University

Dr. Nahed Said Abd El-Latef Ali
Lecturer of Mathematical Statistics
Faculty of Education
Ain Shams University

Dr. Montaser Mahmoud Amein Mohamed
Lecturer of Mathematical Statistics
Faculty of Science
Al-Azhar University

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Supervisors:

No.	Name	Profession	Signature
1.	**Prof. Dr. Mostafa M. Mohie El-Din**	Professor of Mathematical Statistics, Mathematics Department, Faculty of Science, Al-Azhar University.	
2.	**Dr. Nahed Said Abd El-Latef Ali**	Lecturer of Mathematical Statistics, Mathematics Department, Faculty of Education, Ain Shams University.	
3.	**Dr. Montaser Mahmoud Amein Mohamed**	Lecturer of Mathematical Statistics, Mathematics Department, Faculty of Science, Al-Azhar University.	

Acknowledgements

First of all my gratitude and thanks to gracious **Allah** who always helps and guides me. I would like to thank **the prophet Mohamed** “peace be upon him” who urges us to seek knowledge and who is the teacher of mankind.

I would like also to thank the supervision committee:

Prof. Dr. Mostafa M. Mohie El-Din, Professor of Mathematical Statistics, Faculty of Science, Al-Azhar University, for accepting supervision of me, learning me ethics and scientific research assets, his good care of me and my colleagues and for his support, encouragement and continuing guidance during preparing this thesis. He discussed with me many research problems in the seminar of statistics which was held in Faculty of Science, Al-Azhar University.

Dr. Nahed Said Abd El-Latef Ali, Lecturer of Mathematical Statistics, Faculty of Education, Ain Shams University, who provided me with guidance and continuous encouragement. She offered me much of her precious time and provided me with her wisdom and knowledge through many discussions we had.

Dr. Montaser Mahmoud Amein Mohamad, Lecturer of Mathematical Statistics, Faculty of Science, Al-Azhar University, who helped me at the first step in this study through his suggestions for the research problems, valuable instructions, guidance and continuous follow up in this study.

Thanks also are due to **Dr. Osman Mohamed Frege**, Head of Mathematics Department, Faculty of Education, Ain Shams University, and all staff members for providing me with all facilities required to the success of this work.

Many thanks to all my teachers and my fellow companions of scientific research in the school of mathematical statistics, headed by Prof. Dr. Mostafa M. Mohie El-Din.

Finally, I am appreciative to my kind parents, my wife, and my beloved family for their support, patience and continuous encouragement. Finally gratitude and thanks to gracious **Allah** who always helps and guides me.

Mohamed Said Mohamed Mahmoud

Contents

List of abbreviations and symbols

1. *GOS*'s: Generalized order statistics
2. *DGOS*'s: Dual generalized order statistics
3. *pdf*: Probability density function.
4. *cdf*: Cumulative distribution function.
5. *iid*: Independent identically distributed.
6. *BLUE*: Best linear unbiased estimation.
7. *MLE*: Maximum likelihood estimation.
8. *IP*: Informative prior.
9. *NIP*: Non-informative prior.
10. *SE*: Squared error
11. *LINEX*: Linear exponential.
12. *GE*: General entropy.
13. *MCMC*: Markov chain Monte Carlo.
14. *MSE*: Mean square error.
15. ρ: Pearson's product-moment correlation coefficient.

16. $H(.)$: Shannon entropy.

17. $H(.;t)$: Residual entropy.

18. $\overline{H}(.;t)$: Past entropy.

19. $I(.)$: Fisher information.

Summary

Concomitants of ordered random variables have a wide variety of applications in many areas such as selection problems, prediction analysis, double sampling plans, inference problems and information theory. Their importance appears in many fields such as biological, biomedical, physical, industrial and economical disciplines. The aim of this thesis is to study the concomitants of ordered random variables arising from Morgenstern family and its extensions.

The thesis consists of seven chapters:

Chapter 1

This chapter is an introductory chapter. It contains definitions and basic concepts that are used throughout the thesis. A summary of the previous studies is introduced as follows: the different types of ordered random variables and its concomitants, Morgenstern distributions, measures of information and Bayesian analysis.

Chapter 2

In this chapter, the concomitants of case-II of *GOS*'s from Morgenstern distributions and recurrence relations between their moments are studied. Further, we provide the BLUE of the location and scale parameters of the concomitants of order statistics from some distributions.

Some results of this chapter are published in:
Journal of Statistics Applications and Probability, Vol. 3(3), 345-353, 2015.

Chapter 3

Our goal in this chapter is to obtain and study the residual and past entropies of the Morgenstern family for concomitants of ordered random variables. We also consider the characterization results based on the entropy function for concomitants of ordered random variables of residual and past lifetime distributions.

Some results of this chapter are published in:
Journal of Probability and Statistics, Vol. 2015, 1-6, 2015.

Chapter 4

Shannon entropy and Fisher information for concomitants of case-I of GOS's from subfamilies of Morgenstern family when the marginal distributions are Weibull, exponential, Pareto and power function are considered in this chapter. Also, we provide some numerical results of Shannon entropy and Fisher information for concomitants of order statistics.

Some results of this chapter are published in:
Arabian Journal of Mathematics, Vol. 4(3), 171-184, 2015.

Chapter 5

Concomitants of ordered random variables are studied for one and two variables about different subjects such as moments, recurrence relation, uncertainty and so on. We classify this chapter into two parts. In the first part, the joint density of the concomitants of case-I of GOS's for Morgenstern family is proposed and study on the moments of such model is considered. Statistical inferences such as MLE, Bayesian estimation under different types of loss function and Bayesian prediction are obtained for the association parameter of Morgenstern family. Applications of these inferences are presented.

In the second part, the joint densities of the concomitants of case-I of GOSs for exponential and power function subfamilies of Morgenstern family are derived. Statistical inference such as MLE and Bayesian estimation under different types of loss functions based on IP and NIP for the distribution parameters, reliability and cumulative hazard functions are obtained. In addition, Bayesian prediction bounds, Bayes predictive estimator and approximate confidence intervals of the estimators are considered. Applications of these results are given for concomitants of order statistics.

Some results of this chapter are published in:
Mathematical Sciences Letters, Vol. 5(3), 1-12, 2016.

Chapter 6

In this chapter, for Weibull subfamily of Morgenstern family, the joint density of the concomitants of case-I of GOS's is used to obtain the MLE and Bayes estimates for the distribution parameters. Applications of these results for concomitants of order statistics are presented.

Some results of this chapter are published in:
Pakistan Journal of Statistics and Operation Research, Vol. 12(1), 73-87, 2016.

Chapter 7

Finally in Chapter 7, based on extensions of Morgenstern family, we derive the concomitants of different types of GOS's and $DGOS$'s.

Measures of information such as Shannon entropy and Kullback-Leibler divergence are obtained.

Some results of this chapter are published in:
Under submission.

Chapter 1

Introduction

The purpose of this chapter is to present a short survey of some needed definitions and basic concepts of the material used in this thesis.

1.1 Order statistics

Order statistics appear in many parts of statistics and play an important role in applied statistics. They and their moments gained their importance in many statistical problems. They are also more applicable in many engineering fields since in these cases the smallest or the largest future realization of a random variable is important than the mean or the median of the distribution.

Definition 1.1.1 *Let $X_1, X_2, ..., X_n$ be a random sample from an absolutely continuous population with pdf $f(x)$ and cdf $F(x)$, $X_i's$ are arranged in nondecreasing order. Then the smallest of the $X_i's$ is denoted by $X_{(1:n)}$, the second smallest is denoted by $X_{(2:n)}$,..., and, finally, the largest is denoted by $X_{(n:n)}$. Thus $X_{(1:n)} \leq X_{(2:n)} \leq ... \leq X_{(n:n)}$ are called the order statistics obtained by arranging the preceding random sample in increasing order of magnitude, see Arnold et al. [9].*