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On Fuzzy Multicriteria Decision Making Problems

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By

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بسم الله الرحمن الرحيم

" قالوا سبحانك لا علم لنا الا ما علمتنا

انك انت العليم الحكيم "

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Abstract

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The main purpose of this thesis is to study an approach for solving nonlinear goal programming with fuzzy parameters in both objective function and constraints and introducing a parametric study of this method through two different cases.

It presents an approach for solving fuzzy multiobjective linear programming problem as a VOP. Also, it presents a goal programming approach to solve the same problem and it contains a parametric study for the last two cases.

It is devoted to apply the last developed goal approach to Fuzzy continuous cooperative static games ($FCCSG_S$).

Moreover, it presents a decomposition algorithm to determine the bargaining set for the players in the continuous static game. In addition, it contains a formulation for new type of continuous static game.

Key words: Fuzzy Parameters, Fuzzy Multiobjective Linear Programming, Goal Programming, Parametric Study, Cooperative Games.

CHAPTER (V)

Conclusions and Points for Future Research

5.1 Introduction

In this chapter, the most important conclusions that can be extracted from this thesis, will be presented besides, recommendations and suggestions for further study will be introduced.

5.2 Conclusions:

The most important conclusions are summarized in the following:

1-An algorithm for solving nonlinear goal programming problem with fuzzy parameters (FNLGP) has been presented and a parametric analysis has been carried out on it for obtaining the stability set of the first kind through two different cases.

2-The α-decomposition algorithm has been proposed to solve the multiobjective linear programming problem with fuzzy parameters in the objective functions and in the constraints (FMOLP), and to treat it as a VOP. In addition, a parametric study has been introduced for the

latter to get the range of α which corresponds to the results of this study.

3-A goal programming approach has been presented for treating multiobjective linear(nonlinear) programming problems with fuzzy parameters. Moreover, a parametric analysis has been carried out on the (NLGP) problem to obtain the stability set of the first kind. Further more, the range of α values corresponding to the results of the parametric study has been determined.

4-The previous goal programming approach for solving (FMOP) problem has been extended to solve fuzzy continuous cooperative static games ($CCSG_S$).

5-A decomposition algorithm has been presented to determine the bargaining sets for r-player game in which cooperation among the players is possible .

6-A formulation has been illustrated for a new type of continuous static games in which the players are divided into two groups. In the first group each player is cooperative, whereas in the second,the players play as a goal and both are playing according to the min-max criterion.

7-Some numerical examples have been presented to illustrate the previously mentioned algorithm's steps.

5.3. Points for Future Research:

There are several other aspects to be explored in this area which hopefully should be solved in the future:

1- Developing a goal programming approach for solving multiobjective integer nonlinear programming problems with fuzzy parameters.

2- The stability set of the second, third and fourth kinds could be required to continue research in this field.

3- Applying the developed solution approaches to solve different applications.

4- Approaches for solving fuzzy continuous static games should be developed.

5- Other types of cooperative continuous static games can be studied.

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List of Abbreviations

	Notations	Meaning	page
1	MCDM	multiple criteria decision making	1
2	MOP_s	multiobjective optimization problems	2
3	FMOP	fuzzy multiobjective optimization problem.	2
4	MOLP	multiobjective linear programming	3
5	VOP	vector optimization problem	3
6	$MODMP_s$	multiobjective decision making problems	3
7	SOP	scalar optimization problems	5
8	KTCE	kuhn-Tucker condition for efficiency	10
9	KTSPCE	Kuhn-Tucker saddle point conditions for efficiency	11
10	LGP	Linear goal programming	13
11	CSG	Continuous static games	25
12	CCSG	Cooperative continuous static games	26
13	DM	Decision making	28
14	NLGP	Nonlinear goal programming	29
15	FNLGP	Fuzzy nonlinear goal programming	29
16	$MOLPP_S$	Multiobjective linear programming problems	46
17	FMOLP	Fuzzy multiobjective linear programming	47
18	NFMOLP	Non-fuzzy multiobjective linear programming	47
19	HFGP	Hybrid fuzzy goal programming	64
20	NLMOP	Nonlinear multiobjective programming	65
21	SNLP	Single nonlinear programming	66
22	NLGP	Nonlinear goal programming	72
23	FCCSG	Fuzzy continuous static games	89
24	LSFCSG	Large scale fuzzy continuous static	90

		games	
25	NFCCSG	Non fuzzy cooperative continuous static games	91
26	X	$=\{x \in R^n \mid g_r(x) \leq 0,\ r=1,2,...,s\}$	3
27	u_r	Lagrange multiplier	9
28	E	$=\{x \in R^n : Ax \leq b,\ x \geq 0\}$	12
29	w_{ij}	differential weights	12
30	h_{ij}	aspiration levels	12
31	d_i^-, d_i^+	deviational variables	12
32	$\tilde{P}$	real fuzzy number	22
33	$\mu_{\tilde{p}}$	membership function	22
34	$\tilde{\lambda}, \tilde{\beta}$	fuzzy numbers	30
35	L_α	The α-level set	31
36	$M`(x,\beta)$	$\{x \in R^n \mid g_r(x,\beta_r) \leq b_r,\ r = 1,2,\ldots,m\}$	31
37	Ψ^*_i	the optimal value of the attainment problem p_i	33
38	S	the stability set of the first kind of the parametric problem	35
39	c_{ij} , a_{rj}	constants	48
40	e_j	A real value greater than zero	49
41	$M(x, \tilde{\beta})$	$\{x \in R^n \mid g_r(x, \tilde{\beta}_r) \leq b_r,\ r = 1,2,\ldots,m\}$	48
42	V	The solvability set	53
43	θ_{ij}	angle between the gradients of the objectives $f_i(x)$ and $f_j(x)$	68
44	D_{ir}, D_{jr}	the gradient normal vector of the objectives $f_i(x)$,$f_j(x)$	68
45	υ_{ij}	the degree of nonconflict between the objective $f_i(x)$ and $f_j(x)$	69
44	u_i	The control vector	90
45	$G_i(.)$	The cost function of the player i	91
46	Ω^*	$\{u \in R^s \mid h_k(x, u, \tilde{\beta}_k) \geq 0,\ k=1,2,\ldots,m\}$	90
47	P	the pareto –minimal control set.	100

Summary

Multiobjective decision making (MODM) is concerned with the methods and procedures by which multiple objectives can be formally incorporated into the analytical process. The (MODM) elicits in wide variety of problems, such as goal programming, group decision problems (with several criteria), multi-attribute problems, interactive programming etc.

Modeling and optimization methods have been developed in fuzzy environments. In ([24], [25]), Lai and Hwang proposed an interactive fuzzy multiobjective decision model to solve a specific domain of multiple objective decision model. In ([5], [6], [50]), Biswal , Bit and Sakawa presented literatures and papers that deal with the solution of fuzzy multiobjective optimization problems, where many algorithms are suggested for these problems. Also, Sakawa and Yano ([50], [51], [54]), presented an interactive decision making method for multiobjective nonlinear programming problems with fuzzy parameters.

Goal programming is found to be a useful technique for modeling, solving and analyzing multicriteria decision making problems.

In ([13], [33]), Eid and Mohanty described a goal programming approach for solving the multiobjective linear programming problem. Many studies and researches still introduces the concept of Goal programming approaches and its link with multiobjective programming problems and fuzzy set theory.