

MAGNETO - HYDRODYNAMIC DISTURBANCES

A Thesis Submitted

In The Partial Fulfilment of the
Requirements for the Award of the

M. Sc. Degree

BY

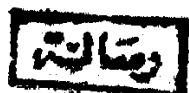
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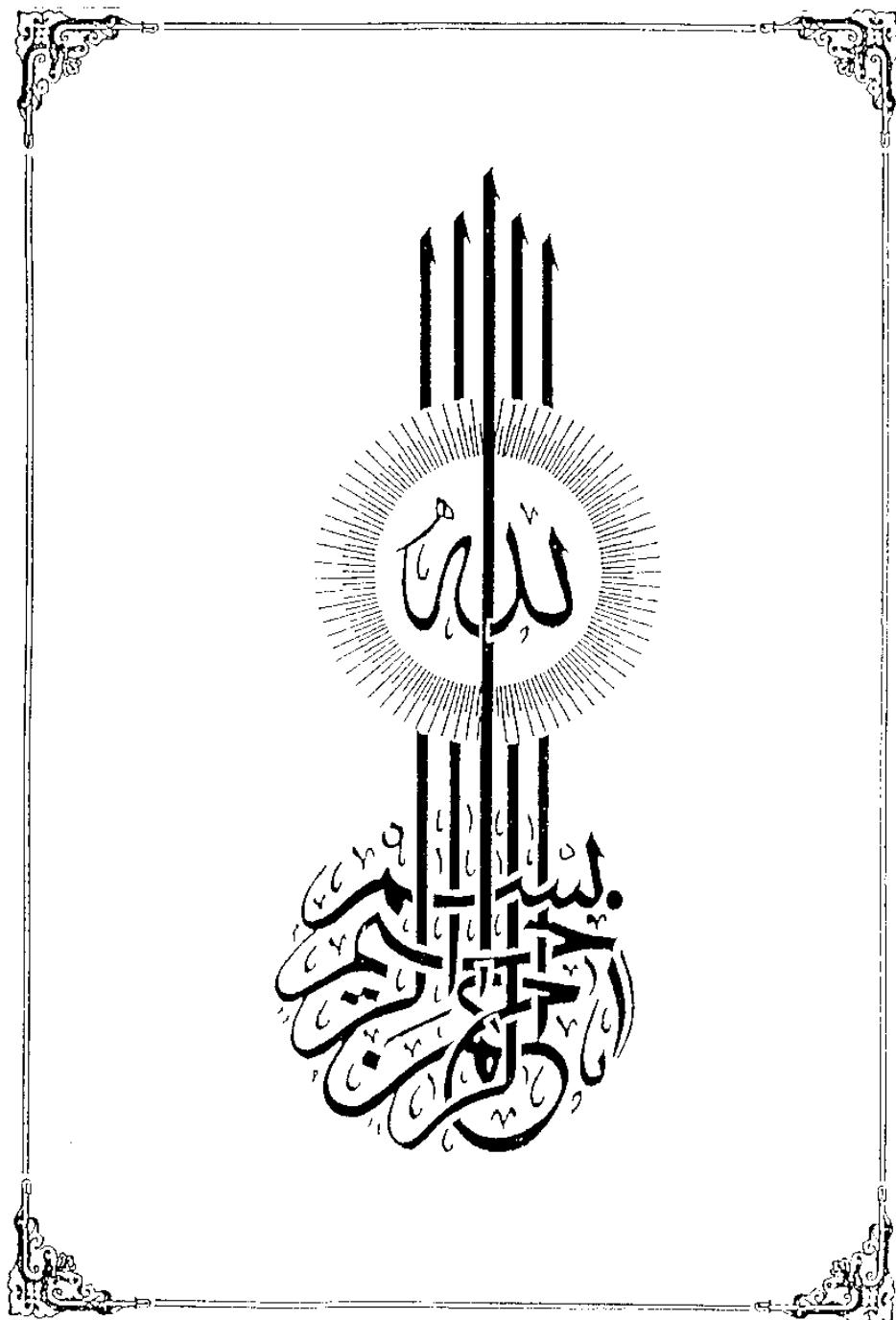
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Summary

The problem of magnetohydrodynamic disturbances in an infinite fluid has been first discussed by P.H. Roberts * (1957) where he has suddenly introduced an infinitesimal current element .

$$\underline{J} = A(t) \quad (r)$$

in a non-viscous fluid of finite electrical conductivity.

M.G.S el Mohandis ** (1959) has extended the problem after that by discussing the hydrodynamic disturbances and fluid motion due to the sudden introduction of a MAGNETIC dipole in an infinite non-viscous fluid in which a uniform field H_0 is prevailing.

The physical importance of Mohandis's work has been discussed by P.H. Roberts and Hide[†] (1961)

* P.H. Roberts, astr. J. no. P.P. (1957)

** M.G.S. el Mohandis, astro. J. 129, 172-193
(1959)

In this work, we discuss the problem of the inclusion introduction of an electric dipole in a non-viscous fluid in which

$\underline{H}_0 = \underline{l}_0$ is taken parallel to the axis of the dipole.

Chapter I discusses an approximate solution of the problem in an infinite medium.

In chapter II the fluid is considered to be a semi-infinite fluid, bounded by insulating plane ; and an approximate solution is obtained only the symmetrical case where \underline{H}_0 is taken to be parallel to the dipole axis and perpendicular to the plane boundary, is considered.

In chapter III, figures representing the approximate results have been drawn.

- + R. Hide and P.H. Roberts : The origin of the Main Geomagnetic Field, ch.2 : physics and chemistry of the earth, vol. 4 (1961), (pergamon press, London).

In chapter IV, an exact solution of the problem for an oscillating dipole in an infinite fluid is discussed for both the symmetric case (\underline{H}_0 parallel to the dipole axis) and for the unsymmetric case ($\underline{H}_0 = H_0 \underline{l}_x$ perpendicular to the dipole axis).

The work of this chapter has been already published ***.

*** Proceedings of the Maths. and phys. Soc., U.A.R.
No-33, 1969 published in 1971 .

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(Approximate Solution)

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EXERCISES : LINEAR INTEGRAL EQUATIONS

(EXERCISES)

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$$\mathbf{H}_0 = \mathbf{H}_0 \mathbf{I}_Z$$

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OPTICAL PROBLEMS IN THE PRESENCE OF A moving ELECTRICALLY CONDUCTING MEDIUM

(Approximate solution)

§ 1.1 INTRODUCTION :

Max well's equations in the presence of a moving electrically conducting matter with velocity \underline{u} and electrical conductivity σ (c.m.s. units used) are :

$$\text{curl } \underline{E} = -\frac{\partial \underline{H}}{\partial t} \dots \dots \dots \dots \dots \dots \dots \dots \quad (1-1)$$

$$\text{curl } \underline{J} = 4\pi \nabla \cdot \underline{J} \dots \dots \dots \dots \dots \dots \dots \dots \dots \quad (1-2)$$

$$\underline{J} = \sigma (\underline{E} + \underline{u} \times \underline{H}) \dots \dots \dots \dots \dots \dots \dots \dots \quad (1-3)$$

also

$$\text{div } \underline{E} = 0 \quad , \quad \text{div } \underline{J} = 0 \dots \dots \dots \dots \dots \dots \dots \quad (1-4)$$

where \underline{E} is the electric field, \underline{H} the magnetic field, \underline{J} the electric current .

Taking the curl of equations (1-2) and (1-3), substituting in equation (1-1) and applying conditions (1-4) we have:

$$\frac{\partial \underline{H}}{\partial t} / \sigma = (1/4\pi\sigma) \nabla^2 \underline{H} + \text{curl} (\underline{u} \times \underline{H}) \dots \dots \dots \dots \dots \quad (1-5)$$

The hydrodynamic equation is

$$\frac{du}{dt} = \underline{F} - \frac{1}{\rho} \nabla p \dots \dots \dots \dots \dots \dots \dots \dots \quad (1-6)$$

where $d/dt = \partial/\partial t + \underline{u} \cdot \text{grad}$ (mobile operator) and the moving matter is considered to be an incompressible, inviscid fluid with density ρ and hydrostatic pressure p . The only force \underline{F} in this case is $\underline{J} \times \underline{H}$ which is the mechanical force exerted by a magnetic field on a volume element of the fluid

carrying a current density \underline{J} also the equation of continuity shows that,

$$\operatorname{div} \underline{u} = 0 \dots \dots \dots \dots \dots \dots \quad (1-7)$$

Equations (1-5) and (1-6) together with conditions (1-4) and (1-7) are the fundamental equations describing the electromagnetic field and its effect on the motion of the fluid.

§ 1.2 BASIC EQUATIONS

Let a source of disturbance be suddenly introduced at zero time in an infinite mass of fluid considered initially at rest and penetrated by a uniform field \underline{H}_0 , according to this sudden disturbance a velocity \underline{u} in a part of the fluid will result as well as a small disturbance \underline{h} of \underline{H}_0 , such that,

$$\underline{H} = \underline{H}_0 + \underline{h} \dots \dots \dots \dots \dots \dots \quad (1-8)$$

$$\underline{h} \ll \underline{H}_0$$

Where \underline{h} always satisfies the condition

$$\operatorname{div} \underline{h} = 0 \dots \dots \dots \dots \dots \dots \quad (1-9)$$

Neglecting squares and products of the small quantities \underline{h} and \underline{u} , the basic equations (1-5) and (1-6) reduce to :

$$\frac{\partial \underline{h}}{\partial t} - (1/4\pi\epsilon_0) \nabla^2 \underline{h} = \operatorname{curl} (\underline{u} \times \underline{H})$$

where \underline{H}_0 is uniform.

But

$$\begin{aligned} & \nabla \cdot \underline{\underline{H}} \cdot (\underline{\underline{H}} \cdot \underline{\underline{H}}) + \underline{\underline{H}} \cdot \underline{\underline{H}} \cdot \underline{\underline{H}} \cdot \underline{\underline{H}} - (\underline{\underline{H}} \cdot \underline{\underline{H}}) \underline{\underline{H}} \\ & + [(\underline{\underline{H}}_0 + \underline{\underline{h}}) \cdot \underline{\underline{H}} \cdot \underline{\underline{H}}] \underline{\underline{H}} - (\underline{\underline{H}} \cdot \underline{\underline{H}} \cdot \underline{\underline{H}}) (\underline{\underline{H}}_0 + \underline{\underline{h}}) \\ & - (\underline{\underline{H}} \cdot \underline{\underline{H}}) \underline{\underline{H}} \end{aligned}$$

where we neglect squares and products of the small quantities $\underline{\underline{h}}$ and $\underline{\underline{u}}$. i.e. neglecting $(\underline{\underline{h}} \cdot \underline{\underline{grad}}) \underline{\underline{u}}$ and $(\underline{\underline{u}} \cdot \underline{\underline{rad}}) \underline{\underline{h}}$ also $(\underline{\underline{u}} \cdot \underline{\underline{rad}}) \underline{\underline{H}}_0 = 0$, $\operatorname{div} \underline{\underline{H}}_0 = 0$, $\operatorname{div} \underline{\underline{u}} = 0$

Therefore

$$\partial \underline{\underline{h}} / \partial t + (1/\mu) \nabla^2 \underline{\underline{h}} = (\underline{\underline{H}}_0 + \underline{\underline{grad}}) \underline{\underline{u}} \dots \dots \dots \quad (1-10)$$

also equation (1-6) reduces to

$$\frac{d\underline{\underline{u}}}{dt} = \frac{1}{\rho} \underline{\underline{J}} + (\underline{\underline{u}} \cdot \underline{\underline{rad}}) \underline{\underline{u}}$$

neglecting a square of $\underline{\underline{u}}$ we have

$$\begin{aligned} \frac{d\underline{\underline{u}}}{dt} &= \frac{1}{\rho} \underline{\underline{J}} + \frac{1}{\rho} \underline{\underline{t}} \\ &= \frac{1}{\rho} \underline{\underline{J}} + \underline{\underline{H}} \cdot (1/\mu) \nabla p \dots \dots \dots \quad (1-11) \end{aligned}$$

using equation (1-2) we have

$$\begin{aligned} \underline{\underline{J}} \times \underline{\underline{H}} &= (1/4\pi) (\operatorname{curl} \underline{\underline{H}}) \underline{\underline{H}} \\ &= -(1/4\pi) [(\underline{\underline{H}}_0 + \underline{\underline{h}}) \wedge \operatorname{curl} \underline{\underline{h}}] \\ &= -(1/4\pi) [\underline{\underline{H}}_0 \wedge \operatorname{curl} \underline{\underline{h}} + \underline{\underline{h}} \wedge \operatorname{curl} \underline{\underline{h}}] \\ &= -(1/4\pi) \underline{\underline{H}}_0 \wedge \operatorname{curl} \underline{\underline{h}} \dots \dots \dots \quad (1-12) \end{aligned}$$

also from vector analysis we have

$$\begin{aligned} \operatorname{grad} (\underline{\underline{h}} \cdot \underline{\underline{H}}_0) &= \underline{\underline{h}} \wedge \operatorname{curl} \underline{\underline{H}}_0 + \underline{\underline{H}}_0 \wedge \operatorname{curl} \underline{\underline{h}} \\ &\quad + (\underline{\underline{h}} \cdot \underline{\underline{grad}}) \underline{\underline{H}}_0 + (\underline{\underline{H}}_0 \cdot \underline{\underline{grad}}) \underline{\underline{h}} \end{aligned}$$

$$= \underline{H} + \text{curl } \underline{h} + (\underline{H}_0 + \text{rad}) \underline{h}$$

$$\text{i.e. } \underline{H} + \text{curl } \underline{h} = \text{grad } (\underline{h} \cdot \underline{H}_0) - (\underline{H}_0 + \text{rad}) \underline{h} \dots \dots \dots (1-13)$$

Substituting from (1-13) in (1-12) we have

$$\mathbf{J} \cdot \underline{H} = (1/4\pi) \text{rad}(\underline{h} \cdot \underline{H}_0) + (1/\pi) \mathbf{J}$$

$$\times (\underline{H}_0 + \text{rad}) \underline{h} \dots \dots \dots (1-14)$$

But

$$\begin{aligned} \text{grad } \frac{1}{2} (\underline{h} + \underline{H}_0)^2 &= \text{rad } \frac{1}{2} [h^2 + 2\underline{h} \cdot \underline{H}_0 + \underline{H}_0^2] \\ &= \frac{1}{2} \text{rad } \underline{h}^2 + \text{grad } (\underline{h} \cdot \underline{H}_0) \\ &+ \frac{1}{2} \text{grad } \underline{H}_0^2 = \text{grad}(\underline{h} \cdot \underline{H}_0) \dots \dots \dots (1-15) \end{aligned}$$

where we neglect $\text{grad } \underline{h}^2$, and $\text{grad } \underline{H}_0 = 0$

Substituting from equation (1-15) in (1-14) we have

$$\begin{aligned} \mathbf{J} \cdot \underline{H} &= (1/4\pi) [(\underline{H}_0 \cdot \text{grad}) \underline{h} - \text{grad } (\underline{h} \cdot \underline{H}_0)] \\ &= (1/4\pi) [(\underline{H}_0 \cdot \text{grad}) \underline{h} - \text{grad } \frac{1}{2} (\underline{h} + \underline{H}_0)^2] \end{aligned}$$

then substituting in equation (1-11) we have

$$\begin{aligned} \frac{\partial \underline{u}}{\partial t} &= (1/4\pi \rho) (\underline{H}_0 \cdot \text{grad}) \underline{h} - (1/8\pi \rho) \text{grad}(\underline{h} + \underline{H}_0)^2 \\ &- \frac{1}{\rho} \nabla p \\ &= (1/4\pi \rho) (\underline{H}_0 \cdot \text{grad}) \underline{h} - \frac{1}{\rho} \text{grad} [(1/8\pi) (\underline{h} + \underline{H}_0)^2 + p] \\ &= (1/4\pi \rho) (\underline{H}_0 \cdot \text{grad}) \underline{h} + p \dots \dots \dots \dots (1-16) \end{aligned}$$

$$\text{Where } \underline{P} = -\nabla \underline{\psi} \dots \dots \dots \dots \dots \quad (1-17)$$

and

$$\underline{\psi} = \frac{1}{2} [(1/8\pi) (\underline{h} + \underline{H}_0)^2 + p] \dots \dots \dots \quad (1-18)$$

which is the total (hydrostatic and magnetic) pressure divided by the density.

Taking the divergence of equation (1-16) remembering that \underline{u} and \underline{h} are each a solenoidal vector, we have everywhere in an infinite medium, with no singularities

$$\begin{aligned} \nabla \cdot (\text{div } \underline{u}) &= (1/4\pi\rho) \text{div}(\underline{H}_0 \cdot \text{grad}) \underline{h} + \text{div } \underline{P} \\ &= \text{div } \underline{P} \\ &= \nabla \cdot (-\sum G_i) = -\nabla^2 \underline{\psi} \\ &= 0 \end{aligned}$$

$$\text{i.e. } \text{div } \underline{P} = -\nabla^2 \underline{\psi} \dots \dots \dots \quad (1-19)$$

§ 1.3 THE SOURCE OF DISTURBANCE

It is supposed that an oscillating magnetic dipole with moment \underline{M} is suddenly introduced at zero time at the origin along the x -axis, to act as a source of disturbance to the above described original configuration of the system.

Putting

$$\underline{M} = \underline{M} \exp(i\omega t)$$