STATISTICAL COMMUNICATION SYSTEMS
THE THRESHOLD CROSSING CORRELATOR

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CONTENTS

(2) The digital correlators: In the digital correlator [1],[2], [19], [20], both of the input signals are quantized before they are delayed and multiplied. The block diagram of this correlator is shown in Fig. (1.b). The polarity coincidence correlator (P.C.C.) [13], [17], [21], [22] is a special case when two quantization levels are used.

(3) The stieltjes correlators: In the stieltjes correlator [4], only one of the input signals is quantized. The functional diagram of this correlator is shown in Fig. (1.c). The relay correlator [illegible] is a special case, where a quantizer of two levels is used.

All the previous types of correlators are based on the implementation of the conventional correlation estimation algorithm, i.e., by performing the three basic sequential operations: delaying one of the signals with respect to the other, multiplication of the delayed and undelayed signals, and then averaging the products over the required observation period. In the direct correlator, the previous operations are performed directly on the signals or their samples. In the digital correlator, the operations are performed on the quantized signals. In the

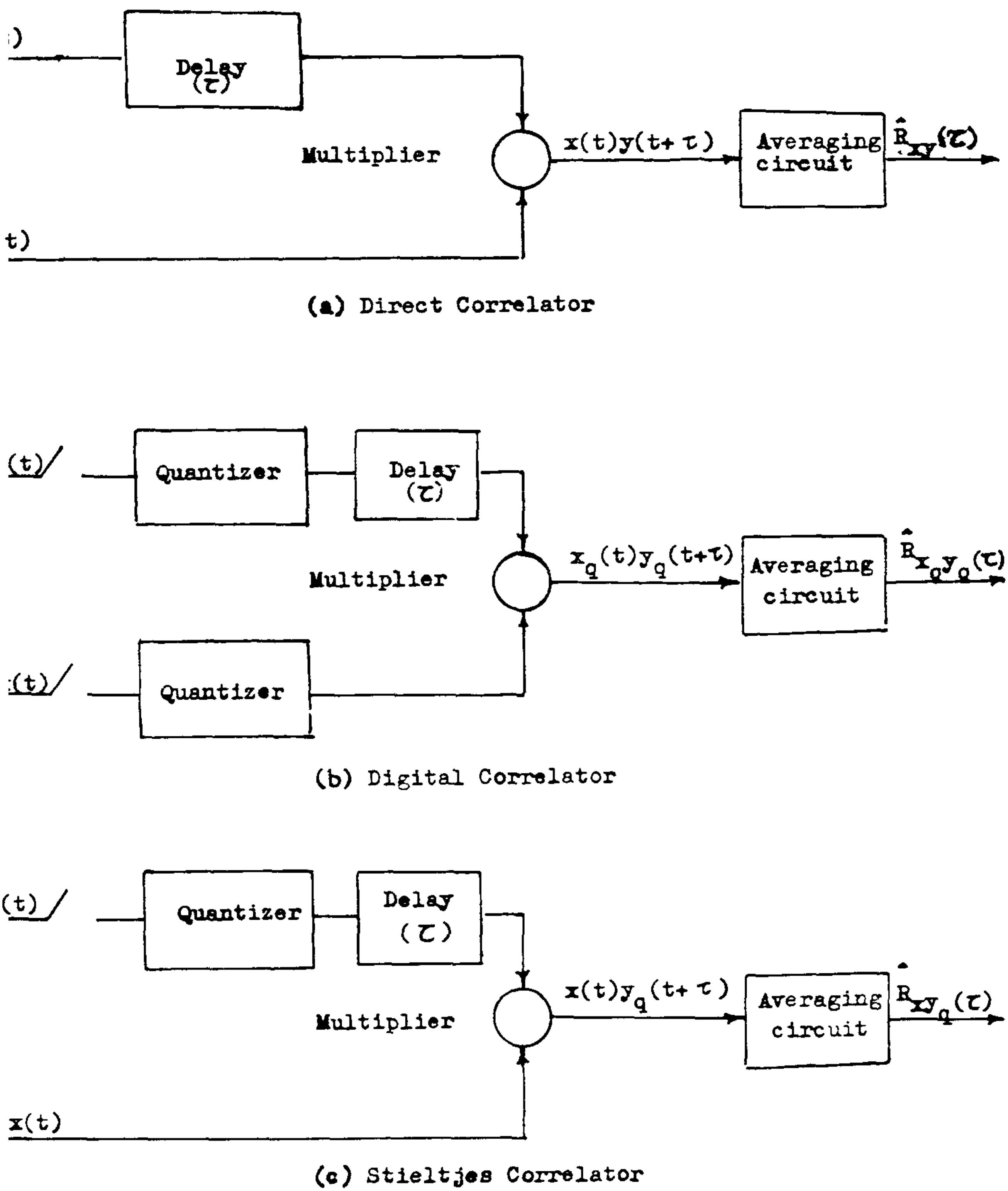

FIG (1) : BLOCK DIAGRAM OF THE CONVENTIONAL CORRELATORS stieltjes correlator the operations are performed on one signal and the quantized version of the other.

Developments related to correlation devices, have two main objectives, first to simplify the implementation, the second is to improve the accuracy of the resulting correlation estimates.

Development in correlation devices related to simplicity in the implementation by reducing demands on processing was studied heavily in the past. In the P.C.C. [5], both signals are infinitely clipped before they are delayed and multiplied. In the relay correlator [13], [20], one signal is correlated against a clipped version of the other. Both types of correlators, have the advantage of constructional simplicity, but their accuracy is worse relative to the multi-level digital, and stieltjes correlators. Other methods were developed to reduce demands on processing, these methods provide inaccurate c.f. measurements utilizing an average response computer. Average response computers (ARC) is the common name for special purpose computers. They compute the average value of the input signal delayed by 'τ' taken over a large number of instants at which synchronization impulses are given. The output is computed for a large number (400 to 1000) of the delay points [25], [26]. The simplest way to implement these methods is to let a sync. pulse be triggered whenever one of the signals y(t) satisfies a certain condition [3], while the other signal x(t) is applied to the ARC. These methods can be categorized as: Dual polarity triggered correlation in which the sync. pulse is generated when y(t) crosses a preset threshold; single polarity triggered correlation, in which the sync. pulse is generated when y(t) crosses a threshold in a specified direction; extreme-value triggered correlation in which the sync. pulse is generated when y(t) passes through a maximum. It was found that the resulting estimates are related to c.f. in the case of Gaussian signals.

Developments in correlation devices related to improving the accuracy of the resulting correlation estimates, were studied in the literature. Such developments are related to improve the implementation techniques of the conventional correlation method. Analogue, digital, or hybrid processing techniques may be used to perform the implementation. These processing techniques were developed to improve the performance of delay circuits, multiplication circuits, and averaging circuits.

Other developments are related to compensate the effect of quantization, when digital processing techniques are used. In this case the resulting estimate is not a true correlation estimate. In the digital correlator, auxiliary noise is added to both signals before they are quantized in order to unbias the output correlation estimate, in this case it is called the modified digital correlator. It is called the modified stieltjes correlator, if only one of the input signals is quantized and one external noise is used. The modified P.C.C. and modified relay correlator [23], [24] are special cases in which the combined signals are quantized into two levels only. The modified multi-level digital correlator, in which uniformly distributed noise is used, is described in [9]. The general form of multi-level modified digital correlator in which any random noise of which the characteristic function has periodic zeros, can be used to unbias the output of the correlator, that is uniformly distributed noise is a special case [4]. It was shown that although the modified P.C.C. has the advantage of constructional simplicity, its error is very large compared with that of the multi-level digital correlator [4].

In this research an efficient method will be developed for estimating c.f. of stochastic signals utilizing the crossing principle. This principle is simple to implement and leads to a threshold crossing correlator. This correlator may be regarded more as a new measuring device for random processes which utilizes an algorithm rather than the conventional correlation algorithm to perform general c.f. computation of stochastic processes.

In the following; chapter one presents the general principle which leads to the new algorithm for the threshold crossing c.f. estimation. It presents how to implement this algorithm into a practical correlator. The c.f. computing arrangement will be deduced. The features of the new correlator are also discussed.

Chapter two, presents general statistical error analysis for our correlation estimator, the bias error, the variance error, and the mean square error will be derived for the general class of stationary random processes.

Chapter three presents an example of band limited stationary Gaussian processes. This example is made in order to put the results obtained in chapter two into a more quantitative form and to make a solid base of comparison between our estimator and the other conventional c.f. estimators. The effect of the number of thresholds, observation period, the band width, and the normalized cross c.f. of the input signals on the accuracy of the new estimator, will be studied through this important example. A study is made for the effect of apriori knowledge of the first order probability density of the reference channel of our correlator in improving the accuracy of the resulting correlation estimate. The mean square error of the estimator will be compared to that of the conventional direct c.f. estimator. The effect of the lower frequency end of a band pass spectrum of the input signals on the accuracy of our estimator will also be derived.

Chapter four, presents analysis of SNR of the threshold crossing correlator. The output SNR of the multilevel crossing correlator is evaluated and compared to that of the conventional direct correlator at various values of the input SNR. More detailed SNR analysis will be given for the single threshold crossing correlator.

CHAPTER 1

General Principle and Implementation of the threshold Crossing Correalation.

In this chapter, the conventional correlation function (c.f.) estimator of stochastic signals is discussed. A new algorithm will be developed utilizing the crossing principle for estimating c.f. of stochastic signals. The resulting correlation estimator is seen to be very close to the true c.f. The implementation of this algorithm into a practical correlator is considered which leads to a threshold crossing correlator. The features of the new correlator compared to those of the conventional correlator are discussed.

1.1. General Principle

Let us assume the two random processes x(t) and y(t) to be stationary, the ensemble cross correlation function $R_{XY}(\tau)$ is given as

$$R_{XY}(\tau) = E\left[x(t) \cdot y(t+\tau) \right]$$

$$= \iint_{-\infty}^{\infty} xy \; f_{XY}(x,y;\tau) \, dx \, dy \quad \ldots\ldots\ldots\ldots \quad (1.1)$$

where 'E' denotes expectation, 'X' and 'Y' are two random variables represented by

$$X = x(t), \; Y = y(t+\tau)$$

$f_{XY}(x,y;\tau)$ is the joint probability density of the random variables 'X' and 'Y', and 'τ' is the relative delay between the two signals x(t) and y(t).