ON NONLINEAR EVALUATION EQUATIONS AND ITS APPLICATION

Thesis

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CONTENTS

SUMMARY

SUMMARY

This thesis is devoted to study the problems of existence and formulation of the variational principles, conservation laws, via invariant variational principles for nonlinear differential equations of physical systems, in combination with a direct variational method is developed for studying the asymptotic behavior of a wide class of nonlinear oscillation and wave problems.

The thesis in all comprises consists of three Chapters and a list of references.

In Chapter I: It is devided into two parts

(i) Abreif survey of the available literature related to the work in Chapters 2 and 3 .

(ii) A brief resine of the consistency conditions for establishing the existence of variational principles for any single or system of two second order nonlinear partial differential equations indicating the procedure for writing down the functional whenever it exists. The classical Noether's theorem with reference to Rund's invariance identities for finding the one-parameter infinitesimal transformation groups is considered. Also this Chapter comprises a short resume about the direct approach which studies the asymptotic behavior of nonlinear oscillation and wave problems due to Hsieh.

In Chapter II: After proving the invariance under a one-parameter infinitesimal transformation groups, the Noether's theorem is then used for writing down the conservation laws for two important systems which describe a physical phenomena represented by

(i) The nonlinear transonic Gas flow equation.

(ii) The nonlinear Schrödinger equation with variable coefficients.

In Chapter III: An approximate, direct method has been applied to deal with the problem of forced oscillation of nonlinear systems. The general prosedure is illustrated in detail and has been applied.

(i) The Mathieu differential equation.

(ii) The problem which describes the theory of synchronization.

(iii) The problem of anti-rolling stabilization.

(iv) A example describes a problem of forces oscillations of nonlinear system.

The same procedure is also applied to some other problems in mathematical physics such as

(v) The Elliptic equation of variable coefficients.

CHAPTER I

INTRODUCTION THE SURVEY AND THE NECESSARY MATHEMATICAL PRELIMINARIES

§(1.1) The survey

In this thesis we shall present a survey and development of the work related with the variational principles and Quasi-variational principles techniques and their applications to nonlinear differential equations which describe a physical and engineering phenomena. For this effect we need to define a functional integral. Functional integral is a rule which associates a real number to each function in some given class of admissible functions. As it is known, a variational principle is a statement which asserts that the functions that solve some equations make a functional integral stationary. However, it is an immediate consequence of the definition of a conservative system the search for a variational principle is equivalent to the inverse problem of the calculus of variations. That is, finding the functional integral whose stationary points are described by the descriptive equations. In fact, a functional integral corresponding to a given differential equation or a system of differential equations was formulated through a practice only. One such attempt was made by Millikan [31] who gave the definitive treatment of the existence of variational principle for the steady-state Navier-stokes equations for an incompressible fluid. He concluded that such an existence is not possible unless.

$$\vec{u}.\nabla.\vec{u}=0 \qquad \text{or} \qquad \vec{u}\times(\nabla\times\vec{u})=0, \qquad (1.1.1)$$

where u is the velocity vector under conditions more general than these conditions, Deshpande [13] established that a two-dimensional incompressible viscous flow past finite bodies can not be equivalent to a variational problem of the Euler-Lagrange type.

In many cases this art of formulating functional integral could not help to find variational principle. Consequently, a systematic approach to the inverse problem of calculus of variations was in order. Though, a straightforward solution to this problem was given by Vainberg [44], Tapia [39] and Nashed [34] but the problem was solved, in principle, by Vainberg in 1954 who gave the analogy between the theory of variational principle and the theory of conservative vector field. It was therefore, proved that the existence of a functional integral for an operator is equivalent to determining whether or not the given operator is potential where a potential operator is the gradient of a functional integral i.e, the operator N is called the potential operator of the functional integral J(u) if $\delta J(u)=\int_{\Omega} N(u)\delta(u)d\Omega$, $\delta J(u)$ is the variation of the functional integral J(u) for a variation δu of the vector u in the direction tangent to a Line [42]. Unfortunately, Vainberg's very general and abstract theorem was still inaccessible to many applied mathematicians, physicists and engineers due to the reason that it was published in Russian. Later on, results were published in English in 1964 [44] Tonti ([42], [43]) recognized the importance of Vainberg's work. Tonti brought Vainberg's work into sharper focus and derived the consistency conditions to determine whether or not a given operator is potential. Tonti used these conditions to obtain the functional integral for a number of physical situations. Finlayson ([17], [16]) used Tonti's formalism and extended the concept of an adjoint operator to nonlinear equations. The cases where the operators failed to become potential operators were examined through the introduction of some integrating factors which transforms these operators to potential operators. Later on, Atherton and Homsy [4] generalized the work of Tonti by rederiving the consistency conditions for the existence of a functional integral for an arbitrary number of nonlinear differential equations in an arbitrary number of independent variables and of arbitrary order in particular, they proved that scalar differential equations of odd order can not be derived through the stationarity of a potential principle. Further, the examples taken up were examined through the following three types of variational principles: potential, alternate potential and composite (adjoint) variational principles which are defined therein. Potential principles are those for which the equations, as written, admit a variational formulation. Alternate potential principles are those for which the equations admit a variational formulation only after a differential transformation of variables. Composite principles are those in which, in addition to the original variables, a set of adjoint variables is defined. The consistency conditions