



*Ain Shams University*  
*Faculty of Engineering*  
*Structural Engineering Department*

Abstract of Ph.D. Thesis submitted by:

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Title of Thesis:

**Buckling and Post Buckling Behavior of Thin Walled Sections  
with Unstiffened Elements under Stress Gradients**

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## **ABSTRACT**

This thesis is an investigation of the behavior of unstiffened plate elements with stress gradients, and thin-walled sections that contain unstiffened elements with stress gradients. The primary aim of this thesis is to produce an accurate design method for thin-walled sections that contain unstiffened elements with stress gradients.

A finite element model is developed to simulate the behavior of such unstiffened plate elements subjected to variety of stress gradients. The finite element model is verified by comparing its results with those obtained from previous research work. The developed finite element model is used to investigate the effect of different parameters on the buckling and post buckling behavior of such plates. The different parameters investigated in the study are plate slenderness, type, and value of the applied stress gradient. Unstiffened plate models are analyzed under stress gradients varying from pure compression to nearly pure bending.

The results of the numerical investigations are used to establish effective width equations for unstiffened elements under stress gradient introducing associated effective stresses to be applied on the effective

width. The effective widths and their associated effective stresses equations are used in conjunction with elastic and inelastic considerations of the effective section, to produce a general design procedure for sections that contain unstiffened elements under stress gradients.

The design method is presented in the form of proposed equations for sections that contain unstiffened elements under stress gradients. The results using the proposed equations are compared to those calculated using different international design specifications and to previous experimental studies on thin walled sections. The results using the proposed equations are found to be in good agreement with test results.



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# **Buckling and Post Buckling Behavior of Thin Walled Sections with Unstiffened Elements under Stress Gradients**

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A Thesis

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To the soul of my father, " I never forget you and your encouragement".

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Ebrahim Abdalla

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# STATEMENT

This thesis is submitted to The University of Ain Shams, Cairo, for the Degree of Doctor of Philosophy.

The work described in this thesis was carried out by the candidate during the years 2004-2010 in the Department of Civil Engineering (structural division) at The University of Ain Shams under the supervision of Pro. Adel Helmy Salem and Pro. Ahmed AbdelSalam El-Serwi.

The candidate declares that the work presented in this thesis is original unless otherwise referenced within the text.

## Conference Papers

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