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SPIN-TORSION INTERACTION AND GRAVITOMAGNETISM

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To My Loving Parents

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Abstract

The present work is devoted to confrontation between predictions of field theories and the corresponding measurements using experiments and/or observations testing such theories. In particular, it is focused on two predictions, the *gravitomagnetism* and *spin-torsion interaction* on one side and the measurements verifying these phenomenae, on the other side. Geometries more wider than Riemannian geometry, have been used to represent such physical phenomenae. Confrontation between predictions and measurements shows that, the sensitivity of the available equipments are not sufficient to test *spin-torsion* interaction. We suggest a space-based experiment, with a more sensitive equipment, for testing such phenomenae.

Keywords: Riemann-Cartan Geometry, Spin-Torsion Interaction, Gravitomagnetism, Flying Clocks, Tests of Gravity Theories.

Summary

The thesis contains three chapters, an Arabic summary, eight figures, seven tables and a list of references.

Chapter 1: TESTS OF RELATIVISTIC THEORIES

This chapter consists of two parts. It is devoted to some tests of relativistic theories. Part I discusses how to use theories to calculate theoretical values for a certain phenomena (predictions of the theory). Part II gives a brief account on some observations and experiments, carried out to test theoretical predictions of the theory (experimental measurements). At the end of this chapter we give a discussion about the material given in the two parts. The discussion shows that geometries, more wider than the Riemannian one, are needed to represent certain gravity interactions.

Chapter 2: GEOMETRIES FOR APPLICATIONS

This chapter, also, contains two parts. It is devoted to geometries suggested to represent physical phenomena, that cannot be represented in the context of Riemannian geometry. In Part I of this chapter, a detailed account of the Parameterized Absolute Parallelism (PAP-)geometry, as an appropriate replacement for