Ain Shams University
University College for Women
Department of Mathematics

SOME OPERATIONS ON GRAPHS

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FOR THE DEGREE
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IN SCIENCE
(PURE MATHEMATICS)
BY

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بسم الله الرحمن الرحيم

(رب اوزعنى ان اشكر نعمتك
التى انعمت على و على والدى و
ان اعمل صالحا ترضاه و اصلح لى
فى ذريتى انى تبت اليك و انى من
المسلمين)

صدق الله العظيم

Ain Shams University
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Mathematics Department

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Title of Thesis:

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ABSTRACT

ABSTRACT

Shereen Adel Abd El Rahman Mousa. Some Operations On Graphs . Doctor of philosophy in Science dissertation of pure Mathematics, University College for women, Ain Shams University.

The main purpose of this thesis is to study some operations on graphs. The thesis is divided into five chapters:

In **Chapter One**, we present some different notions of geometric topology which is useful to follow up in our work.

In **Chapter Two**, we defined the chaotic general tree. The adjacent, incidence, area, volume, edge area, edge volume, area volume matrices will be introduced. The chains of the chaotic general tree will be identified.
The results of this chapter are published in "International journal of pure and applied mathematics", Vol.70, No.5, 711-722,(IF:0.23.), Bulgaria.

In **Chapter Three**, we will introduce the folding of the chaotic general tree and its chains. The variation of the adjacent, incidence, area, volume, edge area, edge volume, area volume matrices of the chaotic general tree under the folding are discussed. The relations between the folding of the chaotic general tree and their chains will be deduced.
The results of this chapter are published in "journal of mathematics research", Vol.3, No. 4, 118-132, Canada.

Chapter Four , we will define a new type of trees called “Tree with knots". The representation of these trees by matrices will be obtained. Some geometric transformations on the new trees are discussed. Also the variations of the matrices under these operations are achieved.
The results of this chapter are accepted for publication in "Advance in pure mathematics “, U.S.A.

Chapter Five is divided into two sections. In section one, we will define the chaotic tree with knots together with its adjacent and incidence matrices. The limit of foldings on it are deduced.The corresponding changes in the adjacent and incidence matrices under these transformations are acheived.

The results of this section are accepted for publication in "Journal of mathematical archive ", Vol. 3, No. 4, 975-982,(I.F.=0.8), India.

In section two, all cases of the connected sum of the tree with knots are induced.

The results of this section are accepted for publication in "Studies in Mathematical Sciences " , Vol. 4, No. 1, Canada.

Keywords: Graph, Chaos, Folding, Retraction, Tree.

SUMMARY