



Mathematics Department



RADIATION FIELD FOR A THREE-LAYERED ROUGH CONDUCTING MEDIUM

A THESIS
SUBMITTED FOR THE AWARD OF
THE DOCTOR OF PHILOSOPHY OF
SCIENCE DEGREE
IN
(Applied Mathematics)

BY

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M. Sc. (Applied Mathematics)

2008

Submitted to

Mathematics Department - Faculty of Science

Ain Shames University

Cairo, Egypt

2013



Mathematics Department



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ACKNOWLEDGEMENT

If it were not for the blessings of Allah, this work would not have been done in the first place. Allah is the first to be praised for everything in all times who helped me to complete this work.

First, I wish to express my sincerest thanks to my main supervisor **Prof. Dr. Samira T. Bishay**, Mathematics Department, Faculty of Science, Ain Shams University, for suggesting the problem treated in this thesis, and for her guidance, support, helpful discussions, motivation and continuous encouragement during my Ph.D. years. I sincerely appreciate the opportunity she gave me for pursuing my doctoral degree under her supervision. She has been a wonderful supervisor who played an important role in my success in my Ph.D. research.

I also wish to express thanks to **Prof. Dr. Osama Abo-Seida**, Head of Mathematics Department, Faculty of Science, Kafr El-Sheikh University, for his support, encouragement and valuable help.

Last, but not least, I wish to express my deepest gratitude to my beloved wonderful **parents** who taught me the value of education. I would like to say that without their love, care, support, and encouragement, the completion of this dissertation would not have been possible. Moreover, without their presence next to me, my journey of life would be hard and boring, and impossible to arrive where I am now.

Kanan Shehata Shoeib

2013

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SUMMARY

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The problem of electromagnetic fields (EMFs) scattering from layered structures with rough boundaries has been the subject of numerous studies dating back to the 1940's. The interest in this problem is mostly because of all natural surfaces are rough, and its applications arise in diverse areas such as measurements in medical, optics, plasma physics, acoustics, geophysics, radar, communications and remote sensing of soil, sea, ocean, and ice surfaces.

The main objective of this thesis was to study and evaluate by using cylindrical polar coordinates (r, ϕ, z) in (3-D), the solutions of the electromagnetic fields scattering from a vertical magnetic dipole (VMD) antenna located in sea with rough interfaces (three-rough layered structures), theoretically.

The model used in the calculations consisted of three-layered structures (air – seawater – ground), the sea is of mean thickness “ a ”, while the air and the ground are infinite upward and downward, respectively, along the z - axis. All regions are infinite sideward and are homogeneous.

The source is a VMD of magnetic moment IS_0 (I is the current flowing in a small loop of area S_0) was embedded in the middle layer (seawater) at a height “ h ” from the ground surface, so the displacement current is very much less than the conducting current and it was neglected. The fields of an infinitesimal dipole are the classic Sommerfeld problem, and the fields expressions in the form of Sommerfeld type integrals (SI) were highly oscillatory and very difficult to evaluate numerically. Therefore, in this work we used the technique which was developed by Long *et al.* [33] to evaluate SI in more accurate and faster way, and to obtain a closed-form expression for the far field.

The form solutions of the far-field (*i.e.*, SI equations) were first expanded as an infinite series. Then, by using the complex image theory [13] we replaced the reflection coefficient by an exponential function which tends to the summation of algebraic series. Then by using the technique [33], which divided the integrand into two factors; a slowly varying part, and rapidly varying part and calculating the stationary phase point, closed-form expressions for the far-field were obtained rapidly in a few easily remembered steps. The condition of the technique [33] was considered, *i.e.*, the slowly varying part in SI has no singularities at the stationary phase point.

This thesis treated three problems of EMF scattering, and a model used in these calculations was taken in three different forms (two-planar interfaces, two-rough interfaces, and one-rough interface). Therefore, the thesis was divided into three main parts as follows:

- **The first part:**

The model was taken with two-planar interfaces (three-planar layered structures). The complete solutions for the far-field in all regions of the model were derived by using the technique [33] with the aid of complex image theory [13]. The physical meaning of these solutions was discussed and the numerical computations were also carried out. Moreover, the results were represented graphically and were in agreement with those mentioned everywhere.

- **The second part:**

The model was extended to include the effects of the variations which occur in both upper and lower interfaces due to the sea waves (three-rough layered structures). The effects of the roughness exercised on the EMFs which were

radiated from a VMD located in the sea were studied by using the perturbation method which used first by Becker [39]. The problem of transition conditions at the upper and lower rough boundaries of the sea were solved in the imaging space of a Hankel transformation. By using the technique [33] and with the aid of the complex image theory [13], closed-form expressions for the far-field in the three regions were derived in details. Besides, we offered some numerical calculations and some diagrams which showed the importance effects of both disturbance parameters χ and s which did not appear in previous studies, and that increase the electric field strength. Also, the physical meaning was discussed.

The third part:

The model was taken with one rough surface (three-layered structures with rough upper surface), i.e., the perturbation applied only in the upper surface. Similar to the second part, a closed-form expression for the far-field in the air region due to a VMD in seawater was obtained. The physical meaning was discussed and some numerical results were given. In addition, the results obtained in this case was compared with the other two cases (two planar surfaces and two rough surfaces) in the air region and represented graphically.

It is worth noting that, the numerical results obtained in this thesis are more accurate and had enabled some previously unanswered questions to be addressed. In addition, the results can be used in modeling a geo-radar operating on layered media and in developing fast and simple interpretation tools, which can deliver quick estimation of the earth structure. Furthermore, the results can be applied to the remote sensing of complex natural stratification, as well as to the simulation of radio-wave propagation in urban environment and can be used with others remote sensing applications.

INTRODUCTION

Electromagnetics is a brief name for the subject that deals with the theory and applications of electric and magnetic fields. Its implications are of fundamental importance in almost all segments of electrical engineering. Limitations on the speed of modern computers, the range of validity of electrical circuit theory, and the principles of signal transmission by means of optical fibers are just a few examples of topics for which knowledge of electromagnetics is indispensable. Electricity and magnetism affect practically in all aspects of our lives, and affect in our daily basis use starting from radios to cell phones to satellite TV. The most important manifestations of electricity are the lightning, the electrical signals through our nervous system which without it we would not be what we are, and light (an electromagnetic wave) which without it there would be no life on Earth. Over the course of the 19th century, the developments on the electromagnetics have been carried out by many scientists, and until now it is still in continuous development. In what follows, we will give a brief on the developments of electromagnetics.

1. Brief History of Early Electromagnetic

Many scientists and engineers have contributed to the early development of electromagnetic theory.

As early as 1820, Hans Christian Oersted had found that when an electric current passed near a compass, the compass needle moved. These showed that there was some form of invisible energy that was coming from the electric current that passed near the compass.

In 1864, James Clerk Maxwell published a remarkable paper describing the means by which a wave consisting of electric and magnetic fields could propagate or travel from one point to another. This was known as Maxwell's Theory of Electromagnetic (EM) radiation.

In the late 1880's, German physicist Heinrich Hertz proved that this theory is correct during a series of careful laboratory experiments. These experiments included placing two small electrodes a short distance apart and generating high voltage sparks between them. When a large electric spark jumped across the gap, Hertz observed that a smaller spark appeared at a second instrument. Therefore, Maxwell's theory which postulated that the electromagnetic energy did travel through the air and cause the second spark are proven, but Hertz saw no future in the devices and theory since the wave only travelled about a meter.

It was not until the last decade of the 19th century that an Italian scientist named Guglielmo Marconi (1874 - 1937) converted these theories into the first practical wireless telegraph system. Marconi well known as "the father of wireless" realized at the age of 19 that radio waves had a future in communication. He studied the works of Hertz and the mathematical conclusions of Maxwell. Soon, he built a transmitter based on the experiments of Hertz and together with Augusto Righi (his professor); he was sending signals across his father's vineyard in Northern Italy. Although Marconi succeeded in his first invention, the Italian authorities weren't interested in it. Hence, he travelled to Britain hoping that they would be interested in wireless communication. On December 12th 1896, he made his first public demonstration of his devices and it was a triumph.

In 1899, Marconi demonstrated his wireless communication technique across the English Channel and it was a success. In another landmark experiment on

December 12, 1901, Marconi demonstrated transatlantic communication by receiving a signal in St. John's New Foundland that had been sent from Cornwall, England. Finally, in 1909, Marconi was awarded the Nobel Prize for Physics for his pioneering work in the use of wireless communication.

By 19th century and until now, the study of electromagnetics had become a crucial topic, where several approaches have been developed with application in several branches, such as radar, remote sensing, communication, underwater acoustics, detection technologies, geophysics, optics, and plasma physics.

2. Propagation over/in planar surfaces

The propagation of the electromagnetic fields (EMFs) problem excited by a dipole source in planar layered media has been studied widely by several investigators because of its useful applications in subsurface and closed-to-the surface communication, radar and geophysical prospecting and diagnostics.

The first analytical solution for the propagation of electromagnetic waves along the planar boundary between the air and the Earth and its relation to wireless telegraphy was carried out by Zenneck [1] in 1907. He showed that a wave, which was a solution of Maxwell's equations, traveled without change of pattern over a flat surface bounding two homogeneous media of different conductivity and permittivity. When the upper medium is air and the lower medium is a homogeneous dissipative ground, the wave is characterized by a phase velocity greater than that of light and a smaller attenuation in the direction along the interface. Furthermore, Zenneck surface wave, as it has been called, is highly attenuated with height above the surface.

In 1909, Arnold Sommerfeld[2]solved the problem of a vertical dipole over a homogeneous ground (half-space), and was the first who obtained the exact solution to the problem in the form of integrals. In an attempt to explain the physical nature of his solution, he divided the expression for the field into space waves (spherical waves) and surface waves. Both parts, according to Sommerfeld, are necessary in order to satisfy Maxwell's equations and the appropriate boundary conditions. The surface-wave part varied inversely as the square root of the range, and it was identified as the radial counterpart of the plane Zenneck surface wave.

For many years it was believed that Sommerfeld surface wave was the predominant component of the field radiated from a vertical antenna over a finitely conducting ground. In 1919, Weyl [3] published a paper on the same subject and obtained a solution similar to that found by Sommerfeld, but without the term of the Zenneck surface wave. In 1926, Sommerfeld [2] returned to the same problem by using a different approach, and concluded that the term of the Zenneck surface wave was not included in the complete solution for the electromagnetic field (in the original work 1909). Accordingly, this work confirmed the correctness of Weyl's solution.

The complete history of the problem is beyond the scope of this work, but suffices to say that independent derivations by Weyl [3], Norton [4] and others have been more accurate representations for the field of the dipole, and confirmed an error in sign in Sommerfeld's paper [2]. In 1941, Norton [5] has developed his formula for the field components to a stage where numerical results can readily be obtained.

In the pioneering work by Wait [6–10], the Sommerfeld integrals (SI) for the electromagnetic field of electric and magnetic dipoles radiating in the layered

region (stratified medium) were evaluated by using asymptotic methods, contour integration, and branch cuts. In 1966, Baños [11] represented a complete perspective of the historical development of the mathematics of the problem.

The electromagnetic fields of vertical and horizontal electric dipoles on or near the boundary between two different media have been well known in terms of closed-form expressions for many years. When the boundary includes a layer of third material with intermediate properties, the properties of the electromagnetic field radiated by a dipole are in general much more complicated.

Numerous approximation techniques were thereafter developed to obtain more accurate and faster results. Generally there are two kinds of solutions. One is asymptotic expansions of the Sommerfeld integrals (SI) and the other one is numerical integration techniques. The asymptotic expansion methods include uniform asymptotic expansions [12], complex image method [13], discrete complex image method [14, 15], steepest descent (saddle point method) [16], stationary phase [16, 17], and so on. The asymptotic methods tend to be less accurate if the distance between the source point and the observation point is not electrically large enough. For the discrete complex image method, if this distance is much larger than the wavelength, the calculated Green's functions are also not accurate. Therefore to calculate the far field from the current on the microstrip patch antenna, some researchers use the reciprocity theory instead of directly using the Green's functions of the multi-layered media. Furthermore, these expansions have been obtained only for simple configurations like two- or three- layer media and the extension of the methods to arbitrary layered media seems to be difficult. On the other hand, due to the highly oscillatory and slowly decay nature of the Sommerfeld integrals (SI), it is difficult to apply a direct numerical integration