ON CONSTRAINT SATISFACTION PROBLEMS

Thesis

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كما أتقدم بجزيل الشكر لأكاديمية البحث العلمى والتكنولوجيا وذلك لدعمها لى فى إتمام هذه الرسالة .

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Summary

The Constraint Satisfaction Problem (CSP) is a powerful and efficient framework for modelling and solving many real world problems. Many problems in artificial intelligence, computer science, engineering, and other disciplines can be naturally represented as constraint satisfaction problems. In a CSP, the goal is to find values for variables such that these values are not violating any constraints that hold between the variables. It often happens that a CSP is over-constrained, thus researchers may seek to partially solve the problem such as in maximal constraint satisfaction problem (Max-CSP). In Max-CSP framework, the aim is to satisfying maximum number of the constraints.

The objective of this thesis is introducing a study on CSPs and Max-CSPs. Larrosa and Schiex ([25],2004) introduced advanced node consistency property NC* to improve the lower bound of branch and bound (B&B) algorithm. Then, Zivan and Meisels in 2007 presented NC*-CBJ algorithm which developed B&B-NC* algorithm without improving its lower bound. During our study, we give a new treatment for improving the lower bound of B&B-NC* and NC*-CBJ algorithms. The proposed treatment leads to

suggesting new algorithm, M-NC*-CBJ which is a natural successor of NC*-CBJ algorithm including the modification of the lower bound. By comparing with the results of NC*-CBJ, the experimental results of M-NC*-CBJ on random CSPs show improvement both in execution times and number of assignments.

This thesis consists of four chapters, conclusion, two appendices, and list of the most recent references used in this study.

The first chapter presents a brief introduction to CSPs. The formal definition of CSP is also given. In addition, classification of CSP according to types of variables and constraints is introduced. Furthermore, this chapter contains the explanation of some problems and how modelling them as CSP. Also, we describe the graphical representation of CSP and how can be exploited to simplify the problem solving. Finally, a brief overview on some practical applications of CSP is included.

The second chapter contains a brief description of the basic techniques for solving CSPs. The proposed techniques are searching strategy, constraint propagation, and combining the search with constraint propagation. Also, some ordering heuristics methods for variables and values in the search strategy are included.

In chapter three, some briefly information about the partial constraint satisfaction problem (PCSP) is introduced. Then, we focus on Maximal Constraint Satisfaction Problem (Max-CSP) which is a special case of PCSP. Also, description of the basic branch and bound algorithm for solving Max-CSP is given. Furthermore, a brief description of NC*-CBJ algorithm which is modified in chapter four, is presented.

Chapter four gives a new treatment for improving NC*-CBJ lower bound, producing M-NC*-CBJ algorithm. This via adding new bound resulting from taking into account more inconsistencies resulted from the partially incompatible relation between the future variables. Also, analysis of complexity of the additional part to NC*-CBJ lower bound and providing a proof that M-NC*-CBJ lower bound is still lower bound are included. Finally, this chapter gives experimental results of M-NC*-CBJ; by comparing with previous results, showing the efficiency and robustness of M-NC*-CBJ and its superiority with respect to the previous NC*-CBJ algorithm. This treatment is considered a new contribution in this field.

In Appendix A, the implementation of M-NC*-CBJ algorithm that is described in the fourth chapter, is written using the programming language C++.

Appendix B is devoted to illustrating the execution of M-NC*-CBJ algorithm on a simple example of CSP. The execution shows that the problem is over-constrained CSP.

Finally, we present a conclusion that summarizes the obtained results and suggest some prospectives for future works.

Chapter 1

Constraint Satisfaction Problem

1.1 Introduction

Have you ever tried to schedule a meeting for a group of busy persons with lots of agenda constraints, solve a crossword or Sudoku puzzle or optimally allocate scarce resources to activities ? If so, you have consciously or unconsciously been dealing with a Constraint Satisfaction Problem (CSP). CSPs are the subject of intense research in both artificial intelligence (AI) and operations research (OR). The regularity of CSP formulation provides a common basis to analyze and solve problems of many unrelated areas, so a large number of problems in AI and other areas of computer science can be viewed as constraint satisfaction problems. Some examples are scheduling, network management and configuration, graph problems, machine vision, database systems, satisfiability problem, the planning of genetic experiments, and the business applications.

In general, a Constraint satisfaction problem (CSP)[40] is a problem composed of a set of variables and a set of constraints. Each variable has a domain of values. Each constraint is defined over some subset of the original set of variables and restricts the values that these variables can simultaneously take. The task of solution is to find an assignment of a value for each variable such that the assignment satisfy all the constraints.

The study of constraint satisfaction problems has been initiated by Montanari in 1974, [28], when describing certain combinatorial problems arising in image-processing. It was quickly realized that the same general framework was applicable to a much wider class of problems. Then, this general problem has since been intensively studied, both theoretically and experimentally (for more information see [27]).

In this chapter, a brief introduction to constraint satisfaction problems is presented. The formal definition of CSP is given in Section 1.2. Classification of CSP according to the types of variables and types of constraints is presented in Section 1.3. In Section 1.4, modelling some problems as CSP is explained. The graphical representation of the CSP and how can exploited to simplify the problem solving are discussed in Section 1.5. Finally, a brief overview of some practical applications of CSP is included in Section 1.6.

1.2 Formalization of constraint satisfaction problem

In this section, a formal definition of the constraint satisfaction problem is given. First, we introduce some terminologies which are used in the formal definition. Almost the information of this part is recalled from [40].

Domains and labels

In a CSP, each variable has a *domain* of possible values, and can only be assigned a value from that domain. Let x be a variable, then its domain is denoted by D_x. Assigning a value to a variable is called labelling a variable. A *label* is a variable-value pair that represents the assignment of the value