Ain shams University
Faculty of Education
Department of Mathematics

Study of some problems on linear and nonlinear hydrodynamic stability and their applications.

A Thesis

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By

Doaa Mohamady Mostafa Elsayed
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Under the Supervision of

Prof. Dr. Mohamed F. El-Sayed
Professor of applied Mathematics
Department of Mathematics
Faculty of Education
Ain Shams University

Prof. Dr. Nabil T. M. Eldabe
Professor of applied Mathematics
Department of Mathematics
Faculty of Education
Ain Shams University

Dr. Mohamed H. Haroun
Associate Prof. of applied Mathematics
Department of Mathematics
Faculty of Education
Ain Shams University

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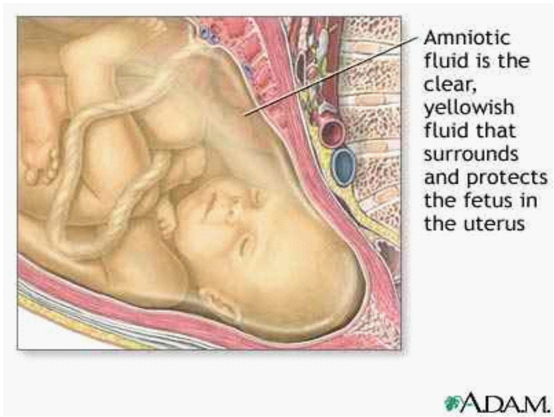
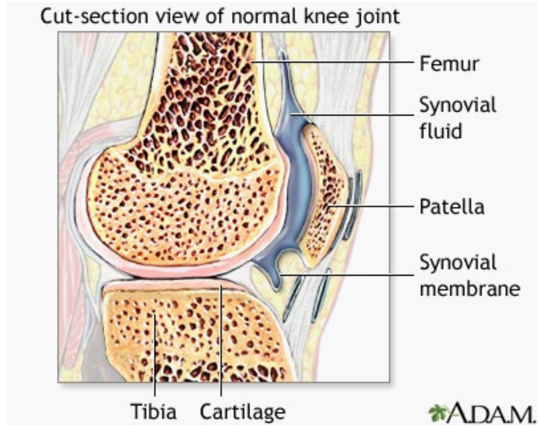
Chapter 1

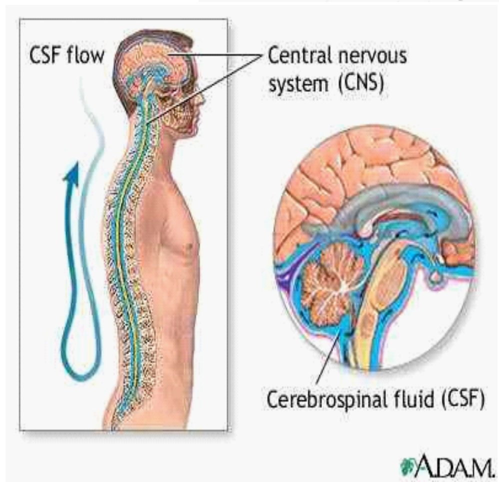
General introduction

1.1 Fluid Mechanics

Fluid Mechanics is a science which deals with the behavior of fluids when subjected to a system of forces and involves the study of kinematics, dynamics and statics of fluids. The term fluid can be defined as substance which deforms continuously due to shear stress. Fluid can be classified into two parts: liquids and gases. The fluid dynamics involves the problems of fluid in different physical conditions in several fields e.g. space science, aerodynamics, rocket propulsion, ship motion on water, power generation by nuclear reactors and hydroelectric power generation etc. The applications of fluid flow are very wide as it plays a very important role in the industries of steel, plastic, electric wire, glasses etc. The system of fluid flow also helps in maintaining the temperature of computer chips, vehide engines and high power machines. In our daily life, lubrication involves the presence of a thin liquid layer that greatly reduces friction and can eliminate squeaks in door hinges, make wheels to turn more easily and prevent engine parts rubbing each other and thus save them from destruction and energy loss.







1.2 Newton's low of viscosity

According to the Newton's low of viscosity, the shear stress on a fluid element layer is directly proportional to the rate of strain, the constant of proportionality being called the coefficient of viscosity. This means that a plot of shear stress versus shear rate at a given temperature is a straight line with a constant slope that is independent of shear rate.

Consider, for example, a fluid contained between two parallel plates, distance h apart, as shown in Fig. (1). If the lower plate is held stationary and the upper plate is moved at a small but constant velocity u by the application of a force F, the fluid will be subject to shearing forces and the velocity will vary across the gap. A typical velocity profile is shown.

Consider the velocity profile, as shown by an enlarged view in the vicinity of a point p. The rate of strain at the point is given by

$$\epsilon = \frac{du}{dy} \tag{1.1}$$

and the shear stresses across the layer are indicated as τ. According to the Newton's Low of Viscosity,

$$\tau \propto \epsilon \text{ or } \frac{du}{dy} \tag{1.2}$$

which may be written as

$$\tau = \mu \frac{du}{dy} \tag{1.3}$$

where the constant of proportionality μ is called the coefficient of viscosity.

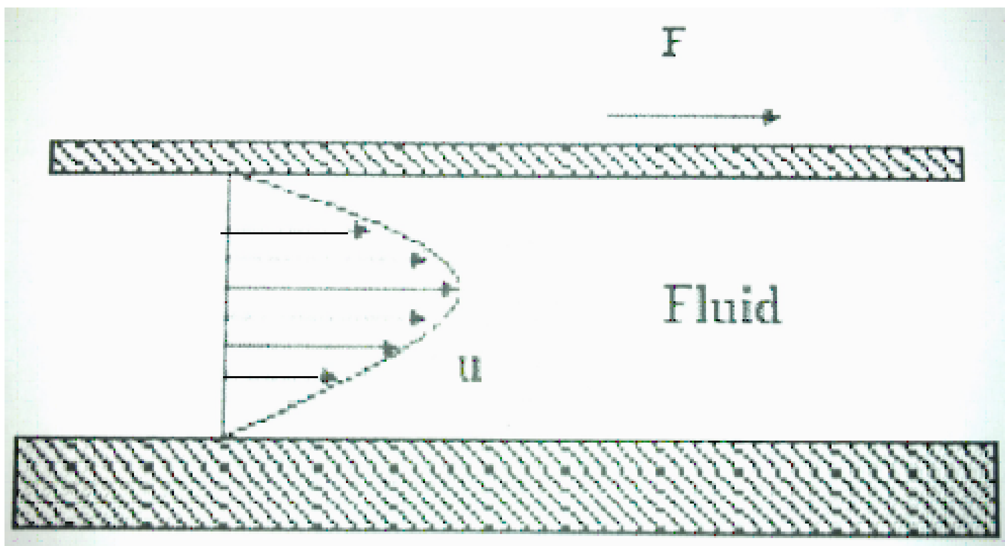


Fig. (1)

Fluids which obey this law are termed Newtonian and those which disobey it are broadly branded as Non-Newtonian. Most engineering fluids such as water, air, mineral oils and molten metals luckily belong to the Newtonian providing relatively easier study. High molecular weight liquids which include polymer melts and solutions of polymers, as wellas liquids in which fine particles are suspended (slurries and pastes), are usually non-Newtonian. In this case, the slope of the shear stress versus shear rate curve will not be constant as we change the shear rate.

1.2.1 Classification of non-Newtonian fluids

Non-Newtonian fluids can be classified as time independent fluids, time dependent fluids and viscoelastic fluids. We summarize them briefly as follows (Chhabra, 2008):

Time- independent non-Newtonian fluids

The viscosity of time independent non-Newtonian fluids is dependent on the shear rate. Depending on how the apparent viscosity changes with shear rate the flow behavior is characterized as follows:

1. **Shear thinning**

The apparent viscosity of the fluid decreases with increasing shear rate. This type of behavior is also referred to as"pseudoplastic" and no initial stress (yield stress) is required to initiate shearing. A number of non-Newtonian materials are in this category includingicaly, milk, gelatin, blood, pllymer, melts such as molten polystyrene, polymer solutions such as polyethylene oxide in water and some paints.

2. **Shear thickening**

The apparent viscosity of this fluid increases with increasing shear rate and no initial stress is required to initiate shearing. This type of behavior is also referred to as "dilatant." Some examples of shear-thickning fluids are clay slurries, sugar in water, corn starch, Beach sand mixed water and solutions of certain surfactants. Most shear-thickening fluids tend to show shear-thinning at very low shear rates.

3. **Viscoplastic fluids**

Viscoplastic materials are fluids that exhibit a yield stress. Below a certain critical shear stress there is no permanent deformation of the fluid and it behaves like a rigid solid. When that shear stress value is exceeded, the material flows like a fluid. Bingham plastics are a special class of viscoplastic fluids that exhibit a linear behavior of shear stress versus shear rate once the fluid begins to flow. An example of a plastic fluid is toothpaste, when you open a tube of toothpaste, it would be good if the paste does not flow at the slightest amount of shear stress. We need to apply an adequate force before the toothpaste will start flowing. So, viscoplastic fluids behave like solids when the applied shear stress is less than the yield stress. Once it exceeds the yield stress, the viscoplastic fluid will flow just like a fluid.

Time-dependent non-Newtonian fluids

For these kinds of fluids, their present behavior is influenced by what happened to them in the recent past. These fluids seem to exhibit a "memory" which fades with time. The apparent viscosity of the fluid depends on a number of properties including shear rate and the history of the shearing process. Depending on how the apparent viscosity changes with time the flow behavior is characterized as"

1. **Thixotropic**

A thixotropic liquid will exhibit a decrease in apparent viscosity over time at a constant shear rate. Once the shear stress is removed, the apparent viscosity gradually increases and returns to its original value. When subjected to varying rates of shear, a thixotropic fluid will demonstrate a "hysteresis loop". Drilling mud, Non-drip paints, tomato ketchup, most honey varieties and cement slurries are among the many materials which can exhibit thixotropic behavior.

2. **Rheopectic**

A rheopectic liquid exhibits a behavior opposite to that of a thixotropic liquid, i,e., the apparent viscosity of the liquid will increase over at a constant shear rate. Once the shear stress is removed, the apparent viscosity gradually decreases and returns to its original value. Rheopectic fluids are rare. Examples include lubricants, specifics gypsum pastes and printers inks.

Viscoelastic fluids

These materials exhibit both viscous and elastic properties. The rheological properties of such a substance at any instant of time will be a function of the recent history of the material and cannot be described by simple relationships between shear stress and shear rate alone, but will also depend on the time derivatives of both of these quantities. Some example of viscoelastic material are polymer melts, bread dough and egg white.

1.2.2 Constitutive equations

Recently the interest in problems on non-Newtonion fluid has grown considerably because of wide use of these fluids in chemical process industries, food and construction engineering, in petroleum production, in power engineering and commercial applications. However, there is not a single governing equation which exhibits all the properties of non-Newtonian fluids and these fluids cannot be described simply as Newtonian fluids. Rheological properties of materials are specified in general by their so-called constitutive equations. Due to this, many constitutive equations for non-Newtonian fluids have been proposed. Constitutive equation are relations between stress and shear rate (rate-of-strain) tensors and can be written for some of non-Newtonian fluids models as

$$\sigma = -pI + 2\mu D(\mathbf{V}) \tag{1.4}$$

where σ is the Cauchy stress tensor, I is the identity tensor, μ is the (not necessarily constant) viscosity, $\mathbf{V}$ is a velocity field and $D(\mathbf{V})$ is the strain rate tensor given by

$$D(\mathbf{V}) = \frac{1}{2}\left(\nabla\mathbf{V} + \boldsymbol{\nabla}^T\mathbf{V}\right) \tag{1.5}$$

The Cauchy stress tensor may be written as two contributions

$$\sigma = -pI + \tau \tag{1.6}$$

where $-pI$: denotes the indeterminate spherical stress and τ is the extra-stress tensor. When the fluid is at rest on a macroscopic scale, no tangential stress acts on a surface. There is only the normal stress i.e., the pressure $-pI$ which is thermodynamic in origin, and is maintained by molecular collisions. The extra-stress tensor τ is due to the relative motion on the continuum scale is called the viscous stress and must depend on gradients of velocity.

The constitutive equations for some models of non-Newtonian fluids are defined as follows:

Time independent fluids

(1) Power-law (Ostwald-deWaele)

The constitutive equation for this model is

$$\sigma = K\dot{\gamma}^{n} \tag{1.7}$$

where K and $\dot{\gamma}^{n}$ are the consistency index for the non Newtonian viscosity and shear rate respectively. The exponent n delineates three cases:

$n < 1$ Pseudo-plastic fluid.

$n = 1$ Newtonian fluid ($K = \mu$).

$n > 1$ Diclant fluids

(2) The Bingham plastic

The constitutive equation for this model is

$$\tau = \tau_B + \mu_{PI}\dot{\gamma}, \qquad |\tau| \geq \tau_B \tag{1.8}$$

$$\dot{\gamma} = 0, \qquad |\tau| < \tau_B \tag{1.9}$$

where τ is the extra stress tensor, τ_B is the yield stress and μ_{PI} is the plastic viscosity. Drilling muds used in the petroleum is an example for this model.

(3) Biviscosity model

Usually Casson model is used as a constitutive equation of blood, but also biviscosity model can be used as a constitutive equation of blood. this model is written as (Nakayama and Sawada 1988):

$$\tau_{ij} = \begin{cases} 2\left(\mu_\beta + \frac{p_y}{\sqrt{2\pi}}\right)\dot{\gamma}_{ij}, & \pi \geq \pi_c \\ 2\left(\mu_\beta + \frac{p_y}{\sqrt{2\pi_c}}\right)\dot{\gamma}_{ij}, & \pi < \pi_c \end{cases} \tag{1.10}$$

where $\beta = \mu_\beta\sqrt{2\pi_c}/p_y$ is yielding stress, μ_β is the plastic viscosity and $\pi = \dot{\gamma}_{ij}\dot{\gamma}_{ij}$, $\dot{\gamma}_{ij}$ is the (i, j) component of the deformation rate.

(4) Casson model

$$\left.\begin{array}{ll} \sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\mu_{PI}\dot{\gamma}}, & \text{for} \quad \tau \geq \tau_y \\ \dot{\gamma} = 0, & \text{for} \quad \tau < \tau_y \end{array}\right\} \tag{1.11}$$

A two parameter model for describing flow behavior in viscoplastic fluids exhibiting a yield response. The parameter τ_y is the yield stress and μ_{PI} is the differential high shear (plastic) viscosity. This equation is of the same form as the Bingham relation, such that the exponent is 1/2 for a Casson plastic and 1 for a Bingham plastic. The casson model has an advantage over the Bingham plastic model for the description of viscoplastic slurry behavior because it can predict curvature of the flow curve at low shear rate, furthermore, the Casson model gives a direct measure of the yield stress and viscosity that is not possible with the three-parameter Herschel-bulkley model. The Casson model has been used to describe the flow behavior of a wide variety of materials, including, printing inks, blood, chocolate and oil-based drilling muds.

(5) Herschel-Bulkley

The Herschel-Bulkley model describes blood behavior. Some examples of fluids behaving in this manner include food products, pharmaceutical products, slurries and semisolid materials. The constitutive equation for this model is

$$\sigma = -pI + 2\left(\mu + \eta\right) D \tag{1.12}$$

where σ is the cauchy stress tensor, $D = \frac{1}{2}\left(L + L^T\right)$ is the symmetric part of the velocity gradient, $L = \nabla V$, T is the transpose, $-pI$ denotes the indeterminate part of the stress due to the constraint of incompressible,μ and η are viscosities.

(6) Carreau - Yasuda

$$\frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \left[1 + \left(\lambda\dot{\gamma}\right)^\alpha\right]^{(n-1)/\alpha} \tag{1.13}$$

A model that describes pseudoplastic flow with asymptotic viscosities at zero (μ_0) and infinite (μ_∞) shear rates, and with no yield stress, the parameter λ is a constant with units of time, where $1/\lambda$ is the critical shear rate at which viscosity begins to decrease. The power-law slope is $(n - 1)$ and the parameter α represents the width of the transition region between (μ_0) and the power-law region. If (μ_0) and (μ_∞) are not known independently from experiment, these quantities may be treated as additional adjustable parameters.

(7) Cross model