



Ain Shams University  
Faculty of Engineering  
Structural Engineering Department

# **BUCKLING BEHAVIOR AND BENDING STRENGTH OF SINGLY-SYMMETRIC OVERHANGING I-BEAMS**

By

**Diana Omar Mohammad Abdul Hameed**

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Supervised By

Prof. Dr.

**Abdelrahim Khalil Dessouki**

Professor of Steel Structures  
Structural Engineering Department  
Ain Shams University

Dr.

**Abdel-Rahim Badawy Abdel-Rahim**

Associate Professor  
Structural Engineering Department  
Ain Shams University  
Cairo 2013

## **Examiners Committee:**

1. Prof. Dr. Adel Helmy Salem (.....)  
Professor of Steel Structures  
Faculty of Engineering  
Ain Shams University
2. Prof. Dr. Nabil Sayed Mahmoud (.....)  
Professor of Steel Structures  
Faculty of Engineering  
Mansoura University
3. Prof. Dr. Abdelrahim Khalil Dessouki (.....)  
Professor of Steel Structures  
Faculty of Engineering  
Ain Shams University
4. Dr. Abdel-Rehim Badawy Abdel-Rehim (.....)  
Associate Professor of Structural Engineering  
Faculty of Engineering  
Ain Shams University

## **STATEMENT**

This dissertation is submitted to Ain Shams University for the degree of Master of Science in Structural Engineering.

The work included in this thesis was carried out by the author in the Department of Structural Engineering, Ain Shams University, from March 2007 to April 2013.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institute.

Date:

Signature:

Name: Diana Omar Mohammad Abdul Hameed

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## ABSTRACT

The design of overhanging singly-symmetric I-beams is not sufficiently covered in the current standards and specifications. The buckling length coefficients specified in the current standards and specifications was firstly defined by Nethercot, 1973 and covers only doubly symmetric I-sections. In this research program, a theoretical analysis is performed to study the elastic and inelastic lateral torsional buckling behavior of singly-symmetric I-sections bent about their major axes.

A finite element model is developed to simulate the behavior of such beams. To include the effect of large displacement on their behavior, a nonlinear geometric analysis is performed. A multi-linear elasto-plastic response of the material is considered. The Newton-Raphson iterative method is used to perform the nonlinear analysis and the load is applied in increments. The results from the developed finite element model showed a good agreement with past experimental results. Once the validity of this model is verified, the model is used to conduct parametric studies to investigate the effect of mono-symmetric ratio, the unsupported length to the radius of gyration ratio and different boundary conditions on the ultimate moment capacity of the overhanging beams. In the theoretical program, the study is divided into two main groups, each group investigating overhanging beams with different boundary conditions and loading positions. The first group is the overhanging singly-symmetric monorail beams subjected to concentrated load at the

bottom flange of the cantilever tip, while the second group is overhanging singly-symmetric floor beams subjected to concentrated point support at the top flange of cantilever tip. The overhanging beams have a constant back-span length while the overhanging parts have different lengths. In case of singly-symmetric overhanging monorail beams, the design of such beams against global buckling is complex due to the nature of loading and poor boundary conditions. The monorail has lateral restraints only at the top flange and therefore has no apparent torsional restraint. In this study, the cantilever tip has two cases of boundary conditions: top flange laterally restrained, and free tip. In case of singly-symmetric overhanging floor beams, the tip of the cantilever had four cases of boundary conditions: top and bottom flanges laterally restrained, top flange only laterally restrained, bottom flange only laterally restrained, and free cantilever tip. The root support was studied under different boundary conditions in both overhanging beam types and the effect of adding stiffener plate with variable depth at root support was also investigated. The cross section considered in the study has different degrees of mono-symmetry. The end support was fully restrained at top flange while bottom flange was horizontally restrained.

A design model based on the results developed from the finite element analysis was proposed. The ultimate moment capacities of such beams computed according to many standards and specifications as well as those computed using the proposed

design model were compared to those obtained from the finite element model in this study. The comparison showed that the proposed design model results had a good agreement with the finite element model results in this study. The ultimate moment capacities obtained using the developed model shows good agreement when compared to the FEM results. The comparison showed that the ultimate moment capacities computed according to the AISC-LRFD Specifications (2010) and BS5950-1:2000, varied from conservative to un-conservative, depending on the overhanging length, degree of mono-symmetry and load location along the beam depth.

**Key Words:** Structural Engineering; Steel; Stability; Lateral Torsional Buckling; Singly-symmetric I-Section; Overhanging Beam.

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