



# **A NEW TRANSMISSION EXPANSION PLANNING MODEL BASED ON AC LOAD FLOW**

By

**Kareem Mohamed Ibrahim Ali**

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
In  
**Electrical power & machines engineering**

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Under the Supervision of

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**Title of Thesis:**

A NEW TRANSMISSION EXPANSION PLANNING MODEL BASED ON AC LOAD FLOW

**Key Words:**

Transmission expansion planning problem, linear programming based model, mixed integer linear programming based model, mixed integer nonlinear programming using AC load flow, comparative study.

**Summary:**

Power system transmission expansion planning is one of the most important studies applied on the power grid; this study performs an important role in estimating the number of new transmission line which will be added to the network to achieve the loading conditions and technical constraints at target year with minimum cost.

More than one model can be used to perform this study. In this thesis we concentrate on a new model based on the AC load flow constraints equations, and compare the get results with the data get from the linear programming and mixed integer linear programming based model.

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## List of symbols

$a_k$	Constant coefficient for new line k
$b_k$	Constant coefficient for new line k
$b_k$	The imaginary part of the series admittance of line k
$C_k$	Capital cost of line
$C_{kj}$	Capital cost of step j associated with line k
$C_g$	Cost of peaking generation system in LE/KVA
$C_e$	Cost of unit energy in LE/KWA
F	Objective function of any form
$f_k$	Cost function of new line k
$g_k$	The real part of the series admittance of line k
$I_k$	Current flow on line k
$I_{maxk}$	Maximum current flow on line k
$I_{mk}$	The imaginary part of current flow $I_k$
$I_{rk}$	The real part of current flow $I_k$
$i$	Inflation rate
$i_g$	Annual charge rate of generation system
$g_i$	Constrained equations on the x variables with $i=1,2,\dots,m$
$\bar{K}$	Large positive value
$k1(i)$	Set of lines (existing and new) connected to bus i
$k2(l)$	Set of lines existing lines found in basic loop l
$k3(l)$	Set of new lines with basic loop l
LE	Set of basic loops having only existing lines

$L$	Number of basic loops
$L_K$	Length of line k
$l_s$	Loss factor
$M$	Total number of lines (existing and proposed).
$M1$	Number of proposed lines
$M2$	Number of existing lines
$N$	Number of buses (where bus N is the reference bus)
$n$	Line life in years
$nc(k)$	Maximum number of circuits associated with line k
$Ns(k)$	The planning period of study (number of steps)
$P_{kh}^l$	Power flow on segment l of step h of new line k
$PL_k$	Annual energy loss cost associated with line k
$PD_k$	Annual cost of demand loss associated with line k
$PI_i$	Active power injected at bus i
$PG_i$	Active power generated at bus i
$PD_i$	Active power demand at bus i
$P_k$	Real power flow on new line k
$P_{100+k}$	Real power flow on line k in the opposite direction
$P_{maxk}$	Maximum allowable power on line k
$P_k^i$	The active power flow on line k from bus i
$P_k^j$	The active power flow on line k from bus j
$q_k^i$	The reactive power flow on line k from bus i
$q_k^j$	The reactive power flow on line k from bus j
$Q_k$	Reactive power flow on line k

$QI_i$	Reactive power injected at bus i
$QG_i$	Reactive power generated at bus i
$QD_i$	Reactive power demand at bus i
$QSi$	Reactive power injected from shunt reactive source existing at bus i, if any.
$r_k$	Series Resistance of line k
$\bar{R}_g$	Reserve factor of generation system
$S_k$	Apparent power flow on line k
$S_{maxk}$	Maximum apparent power flow on line k
$\bar{S}_k$	Annual loss cost for unit power transmitted on line k
$SD_k$	Annual demand cost for unit power transmitted on line k
$S_{kj}^l$	Cost of unit power transmitted on segment l of step j of new line k
$T$	Time in hours
$V_{max}$	Maximum permissible voltage magnitude.
$V_{min}$	Minimum permissible voltage magnitude.
$V_i$	Voltage magnitude of bus i
$V_j$	Voltage magnitude of bus j
$w(h)$	Number of linear segments used for step h
$w(l)$	Number of new lines with basic loop l
$x_{lk}$	Series Reactance of line k per unit length
$X_k$	Power or current flow on line k
$X$	Set of variables of n dimension
$Y_{ij}$	The magnitude of the element $ij^{\text{th}}$ of bus admittance matrix
$Z_k$	Zero-one integer variable related to line k
$Z_{kj}$	Zero – one integer variable associated with step j of line i

$\delta_{ijmax}$	Maximum angle permitted for line ij
$\delta_i$	Angle of voltage of bus i
$\delta_{max}$	Maximum line angle permitted
$\theta_{ij}$	The angle of transfer admittance between bus i and j

# ABSTRACT

Present days power systems have a long stage and highly complicated transmissions systems which cost thousands of millions of dollars. Due to the steady annual growth rate, the expansion planning of these large size networks is a complicated nonlinear optimization problem. The solution of this problem using normal optimization tools has been made using nearly every type of these optimization tools, linear, nonlinear, integer and mixed integer programming techniques. The developed model for the application of each tool has its own merits and demerits. Normally, the DC load flow model has been utilized in all or most of these models.

In this thesis, two mathematical models has been presented to be solved by linear programming technique and mixed integer linear programming technique. The existence of the integer variables in the second case has proved to offer large advantages regarding both the cost function and the constrained equations. The capital cost of new lines is correctly simulated and the voltage loop equations containing new lines have been correctly simulated with the fact that a correct or highly accurate mathematical model has been reached.

In the last part of this thesis the AC load flow equations have been simplified and modified in order to replace DC load flow constrained equations by AC load flow one. This permits the considering of reactive power and bus voltage in the mathematical developed model. The AC load flow equations are the exact equations governing the power flow in the transmission system. Also bus voltage magnitudes constraints and stability constraints can be explicitly considered.

This mathematical model which depends on AC load flow gives the perfect planned network that satisfies all quality constraints imposed on the transmission system with the possible least cost. However, on the other side mixed integer - nonlinear programming technique is relatively difficult and consumes large computational time.