

Menofia University  
Faculty of Science  
Mathematics Department

# VIBRATION SUPPRESSION IN ULTRASONIC CUTTING REPRESENTED BY NON-LINEAR DIFFERENTIAL EQUATIONS

A Thesis

*Submitted to Department of Mathematics, Faculty of Science, Menofia University for  
the Degree of Doctor of Philosophy of  
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By

**Yaser Salah Hamed Hasanien**

*Assistant Lecturer of Mathematics Department of Physics and Engineering Mathematics  
Faculty of Electronic Engineering, Menofia University*

Supervisors

**Prof. Dr. M. H. Eissa**

( )

*Professor of Vibrations*

*Department of Physics and Engineering  
Mathematics, Faculty of Electronic  
Engineering, Menofia University*

**Prof. Dr. M. M. EL-Sheikh**

( )

*Professor of Mathematics*

*Department of Mathematics  
Faculty of Science  
Menofia University*

**Dr. M. M. Kamel**

( )

*Associate Professor of Mathematics  
Department of Physics and Engineering  
Mathematics, Faculty of Electronic  
Engineering, Menofia University*

**Dr. W. A. A El-Ganini**

( )

*Assistant professor of Mathematics  
Department of Physics and Engineering  
Mathematics, Faculty of Electronic  
Engineering, Menofia University*

لجنة الفحص ومناقشة الرسالة:

التوقيع	الوظيفة وجهة العمل	الاسم
( )	أستاذ الرياضيات التطبيقية - كلية الرياضيات - جامعة وترولو بكندا.	Prof Dr. Sue Ann Campbell-1
( )	أستاذ الرياضيات التطبيقية - كلية التربية - جامعة عين شمس	2- ا.د. أحمد يونس غالى
( )	أستاذ الاهتزازات- كلية الهندسة الإلكترونية- جامعة المنوفية.	3- ا.د. مصطفى حسن محمد عيسى
( )	أستاذ مساعد الرياضيات - كلية الهندسة الإلكترونية- جامعة المنوفية	4- د. مجدي محمد كامل

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

سُبْحَانَكَ يَا عَلِيمُ يَا عَلِيمُ يَا عَلِيمُ  
مَا عَلَّمْنَا

إِنَّكَ أَنْتَ الْعَلِيمُ الْعَلِيمُ  
الْعَلِيمُ

صَلَّى اللَّهُ عَلَيْهِ وَسَلَّمَ

سورة البقرة

آية 32

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*Yaser Salah Hamed Hasanien*

# **LIST OF PUBLICATIONS**

## List of publications

1. M. M. Kamel, W. A. A. El-Ganaini and Y. S. Hamed "Vibration Suppression in Ultrasonic Machining Described by Non-Linear Differential Equations" Journal of Mechanical Science and technology (in press).
2. W. A. A. El-Ganaini, M. M. Kamel, and Y. S. Hamed "Vibration Suppression in multi-tool Ultrasonic Machining having to Multi-External Forces" Contemporary Engineering Science (accepted for publication in 1-11-2008).
3. M. M. Kamel, W. A. A. El-Ganaini and Y. S. Hamed "Vibration Suppression in multi-tool Ultrasonic Machining to Multi-External and Parametric Excitations" Acta Mechanica Sinica (online DOI 10.1007/s10409-009-0229-7).
4. W. A. A. El-Ganaini, M. M. Kamel, and Y. S. Hamed "Vibration Reduction in Ultrasonic Machine to External and Tuned Excitation Forces" Applied Mathematical Modeling. 33 (2009) 2853-2863.

**ABSTRACT**

## **Abstract**

The main object of this thesis is the mathematical study of the vibration behavior in ultrasonic machining (USM) described by non-linear differential equations. The ultrasonic machining (USM) consists of the tool holder and the absorbers representing the tools. The coupling of the tool holder and tools simulating ultrasonic cutting process are investigated. This leads to two-degree-of-freedom system subject to multi-external excitation forces in chapter two and a four-degree-of-freedom system subject to multi-external excitation forces in chapter three, both multi-external and multi-parametric excitation forces in chapter four and both external and tuned excitation forces in chapter five. The aim of this work is the reduction of the vibrations in the tool holder and have reasonable amplitudes for the tools represented by the multi-absorbers. The proposed technique allows the USM of multi-shapes at different materials simultaneously. This method improves machine efficiency and saves machining time. Multiple scale perturbation method is applied to obtain the solution up to and including the second order approximation and to study the stability of the steady state solution near different simultaneous resonance cases. The resulting different resonance cases are reported and studied numerically. The stability of the steady state solution near the selected resonance cases is studied applying both frequency response equations and phase-plane technique. The effects of the different parameters of the system and the absorbers on the system behavior are studied numerically. Optimum working conditions for the tools were obtained. Comparison with the available published work is reported.



# ENGLISH SUMMARY

## Summary

Vibrations and dynamic chaos are undesired phenomenon in structures. They always cause disturbance, discomfort, damage and destruction of the systems or the structures that represent many engineering applications. One of the important subjects expressed here is the dynamical systems, which are governed by different types of non-linear differential equations. Sometimes vibrations are desirable as for the tool in the ultrasonic machining (USM), as the machining technique is dependent on tool and abrasive particles vibration. In such case the problem is a complicated one, as it is required to reduce the vibration in the tool holder and have reasonable amplitude for the tools (absorbers). This can be employed via passive and active control methods.

The main object of this thesis is the mathematical study and suppress the vibration behavior in ultrasonic machining (USM) described by non-linear differential equations. The ultrasonic machining (USM) consists of the tool holder and the absorbers as the tools. The aim of this work is the reduction of the vibrations in the tool holder and have reasonable amplitudes for the tools represented by the multi-absorbers. This can be done via passive or active control techniques. This work is limited to passive control only by adding the multi absorbers (as a tools) to control the vibration due to tool holder. The proposed technique here allows the USM of multi-shapes at different materials simultaneously. This method improves machine efficiency and saves both machining time and tool holder life.

Two and four degree of freedom system of non-linear differential equations are considered and studied. For each considered system the investigation includes the analytical solutions applying the multiple scales perturbation technique (MSPT), determination of the resonance cases,

which are classified into different categories. Some of reported resonance cases are confirmed numerically. The stability of each vibrating system and its steady state solutions are investigated by using both the frequency response equation and phase-plane technique. The numerical solutions are focused on both the effects of the different parameters and the behavior of the system at the selected resonance cases. By the end of the work a comparison is made with the available published previous work. Some recommendations are given regarding the design of such system.

Our thesis is divided here into six chapters:

## **Chapter One**

This chapter deals with the objective of this work and literature review.

## **Chapter Two**

This chapter includes both analytical and numerical solutions of the two-degree of non-linear differential equations that describe the vibration of ultrasonic machine subjected to multi-external excitation forces at simultaneous primary and internal resonance case.

## **Chapter Three**

In this chapter we study analytical and numerical solutions of the four-degree-of-freedom non-linear differential equations that describe the vibration of ultrasonic machine subjected to multi-external excitation forces at simultaneous primary and internal resonance case.

## **Chapter Four**

This chapter reports both analytical and numerical solutions of the investigated system of non-linear differential equations that is mentioned in chapter three but under both multi-external and multi-parametric excitation forces. The investigated system is studied and solved at the simultaneous primary, sub-harmonic and internal resonance case.

## **Chapter Five**

In this chapter we report both analytical and numerical solutions of the investigated system of non-linear differential equations that is mentioned in chapter three but under both external and tuned excitation forces at the simultaneous primary, combined and internal resonance case.

## **Chapter six**

This chapter demonstrates the main results and conclusions that obtained in this work, due to both analytical and numerical studies. A comparison is made with available published works and suggestions for future work are demonstrated.

# **NOMENCLATURE**

## Nomenclature

$\left. \begin{array}{l} A_n, (n= 1, 2, 3, 4) \\ B_m, (m= 1, 2) \end{array} \right\}$	Complex functions in $T_1, T_2$ .
$\left. \begin{array}{l} a_n, (n= 1, 2, 3, 4) \\ b_m, (m= (1, 2)) \end{array} \right\}$	The steady state amplitudes of the tool holder ( $\mu m$ ).
$\left. \begin{array}{l} \psi_1, \psi_2, \phi_1, \phi_2 \\ \beta_n, (n= 1,2,3,4) \end{array} \right\}$	The phases of the motion.
$D_0, D_1, D_2$	Differential operators; $D_n = \partial / \partial T_n$ ( $n = 0,1,2$ ).
$t, T_0$ and $(T_1, T_2)$	Time , fast and slow time scales (Sec).
$\left. \begin{array}{l} c_i, (i= 1,2,3,5) \\ \zeta_k, (k= 1,2,3,8) \end{array} \right\}$	Damping coefficients of the tool holder and the tools ( $Ns / \mu m$ ).
$\left. \begin{array}{l} \eta_s, (s= 1,2,3,7) \\ k_n, h_n, (n= 1,2,3,4) \end{array} \right\}$	The non-linear parameters of the tool holder and tools ( $N / \mu m$ ).
$F_j, \Omega_j, (j=1, 2, \dots, n)$	Excitation force amplitudes ( $\mu m / s^2$ ) and excitation frequencies (Hz).
$\omega_n, n= (1, 2, 3, 4)$	Natural frequencies (Hz).
$x_n, n= (1, 2, 3, 4)$	Axial displacement functions ( $\mu m$ ).
$\dot{x}_n$ and $\ddot{x}_n, n= (1, 2, 3, 4)$	Response first derivatives (velocity) ( $\mu m / s$ ) and second derivatives (acceleration) ( $\mu m / s^2$ ) w.r.t t.
$\varepsilon$	Small perturbation parameter.
$\lambda_n, n= (1, 2, \dots, 8)$	Eigenvalues.
$\sigma_n, n= (1, 2, 3, 4)$	Detuning parameters.

## NOMENCLATURE

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$E_a$	Effectiveness of the tool ( $\mu m$ ).
$\theta_n, (n = 1, 2, 3, 4)$	Phase angle of the polar forms.
$k_n, (n = 1, 2, \dots, 8)$	Complex functions in $T_1, T_2$ .
$K_n, (n = 1, 2, \dots, 36)$	
$H_s, (s = 1, 93, \dots, 177)$	
$E_s, (s = 1, \dots, 798)$	
$z_i, (s = 1, \dots, 176)$	
$Q_i, (i = 1, \dots, 264)$	
$(\Gamma_1, \Gamma_2, \dots, \Gamma_{34})$	Real constant.
$m_1, m_2, m_2$ and $m_4$	The mass of the tool holder and tools respectively.