



Faculty of Science

Mathematics Department

# Evolution of Strategies in the Two and Three Players Iterated Games

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## A Thesis

Submitted for the degree of Master of Science as a partial fulfillment for  
requirements of the Master of Science

**(Pure Mathematics)**

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**2017**



# *Acknowledgment*



## *Acknowledgements*

I would like to thank all those people who made this thesis possible and an unforgettable experience for me. First of all, I would like to express my deepest sense of Gratitude to my supervisor Prof. Essam El-Seidy who offered his continuous advice and encouragement throughout the course of this thesis. I thank him for the systematic guidance and great effort he put into training me in the scientific field.

I would like to express my very sincere gratitude to Prof. Entisarat M. Elshobaky and Prof. Salah El Din S. Hussein for the support and encouragement whenever I was in need to make this thesis possible. I am thankful to Mr. Abdullah Ahmed for his precious assistant during coding all of algorithms in my thesis. I also want to thank Mrs. Marwa Hossam El Din for her flawless grammatical editing of my thesis.

Finally, I take this opportunity to express the profound gratitude from my deep heart to my beloved parents, brothers, and my friends for their love and continuous support both spiritually and materially. I also would like to give my gratitude to my lovely wife for the support and encouragement whenever I was in need to make this thesis possible.



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# *Abstract*