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**EXTENSIONS OF THE EXPECTATION
- MAXIMIZATION (EM) ALGORITHM
USING A BAYSIAN APPROACH**

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

﴿ وَقُلْ اَعْمَلُوا فَسَيَرَى اللَّهُ عَمَلَكُمْ وَرَسُولُهُ وَالْمُؤْمِنُونَ وَسَتُرَدُّونَ اِلَى

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LIST OF ABBREVIATIONS

AC	: Available – Case Method
AECM	: Alternating Expectation Conditional Maximization algorithm
CC	: The complete Case Analysis
CM	: Conditional Maximization
ECM	: Expectation – Conditional Maximization
ECME	: Expectation – Conditional Maximization Either
EM	: Expectation-Maximization
E-PX	: Expectation step for Parameter Expansion
E-step	: Expectation Step
GEM	: Generalized EM
MAR	: Missing At Random
MCAR	: Missing Completely At Random
MI	: Multiple Imputations
ML	: Maximization Likelihood
MLE	: Maximization Likelihood Estimation
M-PX	: Maximization step for Parameter Expansion
M-step	: Maximization Step
NMAR	: Not Missing At Random
N-R	: Newton – Raphson
PX-EM	: Parameter Expansion to accelerate EM
SAGE	: The Space Alternating Generalized