شبكة المعلومات الجامعية

التوثيق الإلكتروني والميكروفيلم

بسم الله الرحمن الرحيم

HANAA ALY

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جامعة عين شمس

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zero determinate strategies for Iterated Games

A Thesis

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Abstract

Zero Determinant (ZD) strategies is a new class of probabilistic and conditional strategies discovered by Press and Dyson which has been applied on two players. In this thesis, I am interested in applying ZD Strategies in three - player game. Where each player has two pure actions to cooperate or defect, the action in the current state depends only in the previous action in the last state. Also, the transition matrix when one player move from one state to another was determined in case of three players. ZD strategies for multiple players will be explored where the application of ZD has been taken on Iterated three-player Prisoner's Dilemma game (IPD) with two actions Cooperate(C) or Defect(D). The results in case of the three players correspond to what was obtained in case of two players with different calculation methods. Surprisingly, although the player who adopts ZD strategies can determine the opponents' payoffs in a unilateral way, and with an extortion strategy, he can impose a linear relationship between the expected return for him and the expected payoff of other players, but he cannot determine his own score because his technique is leads to one possible strategy (1,1,1,1,0,0,0,0) which is not valid strategy. Then, a player who uses ZD strategies can't be able to set his expected payoff. We show when three players face each other how the player who uses ZD strategies determines the return of one of his/her opponents greater than the return of the others and also ensures that he receives a greater return from all the opponents. Finally, we will display the required conditions by which a player can determine the minimum and maximum returns for opponents.

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List of Abbreviations

PD	Prisoners Dilemma
IPD	Iterated Prisoners Dilemma
C	Cooperate
D	Defect
IPDG	Iterated Prisoner’s Dilemma Game
ZD	Zero Determinate

Summary

The thesis topic is related to the Zero Determinant (ZD)strategies, which were concluded in research published in 2012 by researchers Press and Dyson. A player using these strategies can determine the return of the other player between mutual cooperation and mutual defection values regardless of the strategy the opponent uses. Also, a player using ZD strategies can impose a linear relationship in which the return can be greedily distributed.

The aim of this thesis is to apply ZD strategies in the iterated prisoner's dilemma (IPD) when three players face each other. Show how a player using ZD strategies can dominate the game and control the opponents' payoff regardless of the strategy used by the other players, even though he cannot determine the payoff for himself. Also, showing the possibility of using the extortion strategy when three players faced each other. Presenting the necessary conditions for choosing a strategy through which it determines the minimum and maximum returns of the opponents, as well as the conditions for choosing an unbeatable strategy from the rest of the opponents. This thesis consists of four chapters as follows:

In chapter one: the basic definitions of game theory and the concepts that were used within the thesis. Presenting game types in their normal and extensive form, and referring to basic concepts such as Nash equilibrium, simple and compound strategy, and the return function. Presenting the most famous examples in game theory, such as the prisoner's dilemma, the battle of sexes, and Hawk-Dove game. There is a presentation of some conflicts and their division into cooperative or non-cooperative and the study of utility function according to the circumstances faced by the player.

In chapter two: The strategies of the ZD strategies were presented on the iterated prisoner's dilemma in the stochastic game when two players faced each other. Presenting the transition matrix that shows the player's transition from one state to another. Show all the advantages that the player who uses the ZD strategies can obtain and enable him to control the game as well as control the returns and clarify the necessary conditions to achieve these strategies.

In chapter three: We presenting ZD strategies when three players face each other in the iterated prisoner's dilemma. Where each player has two pure strategies, cooperation or defection, for one player will not affect in his strategy any of the other two players cooperate and any of them split, where each player expands on vector with 8-tuple dimensions with six independent variables. The transition matrix between three players is shown when the players selection changes from cooperation to defection and vice versa. We consider an iterated prisoner's dilemma game where players meet multiple times and may all develop their own strategies depending on the opponents' strategy and history of the game. The results obtained were achieved by assuming the use of one of the players, let it be "X", ZD strategies. Where X can assign the expected results of the opponents between the values of mutual cooperation and mutual defection. If a player using Zero-Determinate strategies tries to develop a strategy to determine his expected return, this leads to one possible strategy (1, 1, 1, 1, 0, 0, 0, 0) making the transition matrix equal to zero then this may not be able to determine its expected return. Finally, a player using ZD strategies can enforce a linear relationship for an extortionate share of payoff with a given range of $\mathcal{X}$ and the expected payoff is calculated for each player.

In Chapter four: Presenting the necessary conditions for the strategy used by the player who uses ZD strategies on the iterated prisoner's dilemma game to obtain an indomitable strategy from other players. A player who is familiar with ZD strategies can also control the minimum and maximum returns of his/her opponents by applying some necessary and sufficient conditions in the strategy used by the player.

Chapter 1: Basic Definitions and Notations

1.1. Markov Chain

We will present a review of Markov process, Markov chain, and focus attention on some definitions and theories specifically in the discreet time Markov chain and its long run behavior. The most of facts that will be described depends on Karlin and Taylor [1], [2].

Markov chains are an important mathematical tool in stochastic processes which are a fairly common, and relatively simple method for describing stochastic processes in a statistical model. They have been used in many different domains, ranging from text generation to financial modeling. Markov chain is one of the techniques to perform a stochastic process the underlying idea is the Markov Property, that is based on the present state to predict the future state of the past states are irrelevant for this stochastic process.

1.2. Discrete Time Markov Chain

Definition 1.1. A stochastic process is a family of random variables X_n where n is a parameter running over a suitable index set T, for convenient, we will write X(n) instead of X_n. the index n corresponds to discrete units of time, and the index set is

$T = \{0, 1, 2, \ldots\}$

Definition 1.2. Markov processes is a stochastic process when given its current state, The future process is determined by its most recent values, which used to model systems that have a limited memory of their past. In other words, when the current state is known exactly then the probability of any future process not change by additional knowledge related to its previous behavior. A typical random process $X = \{X(n): n \in T\}$ is a family of random variables indexed by elements of some set T, X(n) the state of the process at time n.

Definition 1.3. Markov Chain is a Markov process described as following: a set of state space $T = \{0, 1, 2, \ldots\}$ which is finite or countable, called "discrete-time Markov chain", alternatively, for

$T = R$ or $T = [0, \infty)$ one has a "continuous-time Markov Process". Below we will only consider discrete-time processes.

Definition 1.4. A discrete-time Markov chain $\{X_n\}$is a Markov stochastic process whose state space is a finite or countable set, and for which T = {0, 1, 2, ...} is (time) index set.
Formally, the Markov property is that

$$\Pr(X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \ldots, X_0 = i_0) = \Pr(X_{n+1} = j | X_n = i) \quad (1)$$

i.e., the state at time n+1 merely depends on the state at time n.

Definition 1.5. one-step transition probability is the probability of transitioning from one state to another in a single step. Formally, the probability of X_{n+1}, being in state j given that X_n is in state i is called the (one-step transition probability) and is denoted by $P_{i^n j^{n+1}}$. That is,

$$P_{i^n j^{n+1}} = \Pr(X_{n+1} = j | X_n = i) \quad (2)$$

the notation ensures that, the general transition probabilities are function on initial state, final states and time of the transition. When one - step transition probabilities do not dependent on the time variable n, we say that Markov have stationary transition probabilities and is called time-homogenous and in this case $P_{i^n j^{n+1}} = P_{ij}$ is independent on n. We will pay attention to time-homogenous Markov chain because the game we interested in is not time dependent. In this case P_{ij} is the conditional probability that the state value transition from i to j in one round. It is usually to arrange these numbers P_{ij} in a matrix.

$$\mathbf{P} = \begin{Vmatrix} P_{00} & P_{01} & P_{02} & \cdots \\ P_{10} & P_{11} & P_{12} & \cdots \\ P_{20} & P_{21} & P_{13} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{Vmatrix}$$

Where $\mathbf{P} = \|\mathbf{P}_{ij}\|$ called the Markov matrix or the one-step transition probability matrix, or just transition probability matrix. A Markov process is completely determined by transition probability matrix and the initial state. If the number of states finite, then **P** is a finite square matrix whose order equal to the number of states. The