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$\Sigma\Delta$ Analog-to-Digital Converters A Thesis

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Degree in Electrical Engineering

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To my wife
for her incredible tolerance and support

Contents

List of Tables	vii
List of Figures	ix
Abstract	xiii
1 Introduction	1
1.1 Motivation	1
1.2 Outline / Thesis Organization	1
2 Continuous-Time $\Sigma\Delta$ Modulator: An Overview	3
2.1 Analog-to-Digital Conversion: Fundamentals	3
2.1.1 Sampling	4
2.1.2 Quantization	5
2.2 Oversampling $\Sigma\Delta$ ADCs: Fundamentals	8
2.2.1 Oversampling	8
2.2.2 Noise Shaping	9
2.2.3 Basic Architecture of Oversampling $\Sigma\Delta$ ADCs	11
2.2.4 Performance Metrics	13
2.2.5 Ideal Performance	15
2.3 Single-Loop $\Sigma\Delta$ Architectures	16
2.3.1 Loop Filter Architectures	16
2.3.2 NTF Zeros Optimization	18
2.3.3 Stability of Higher Order $\Sigma\Delta$ Modulators	19
2.4 Multi-bit Architectures	21
2.5 Performance Comparison of Traditional $\Sigma\Delta$ Topologies	22
2.6 Continuous-Time versus Discrete-Time Implementations	23
3 Continuous-Time $\Sigma\Delta$ Modulator: System Design	27
3.1 ADC Target Specifications	27
3.2 Power Efficient Noise Budgeting	28
3.3 NTF Selection	29
3.3.1 <i>NTF</i> design space	29
3.3.2 $\Sigma\Delta$ Design and Analysis Toolbox	31
3.3.3 <i>NTF</i> design methodology	33

3.4	DT-to-CT Conversion	39
3.4.1	Impulse-invariant transformation	40
3.4.2	Feedback Pulse Shape Selection	41
3.4.3	Conversion of selected $H(z)$ to $H(s)$	44
3.5	Architecture Selection	47
3.5.1	CRFB and CRFF : Comparison	50
3.5.2	CRFF realization	55
4	Continuous-Time $\Sigma\Delta$ Modulator: Top-Down Design	59
4.1	Signal-Flow Simulations	59
4.2	Noise-based components selection	62
4.2.1	Coefficients to circuits mapping	63
4.2.2	Noise Analysis	66
4.3	Modelling and Analysis of Block Non-idealities	78
4.3.1	Filter Non-idealities	80
4.3.2	DAC Non-idealities	86
4.3.3	ADC Non-idealities	89
4.4	Summary	91
5	Circuit Implementation and Verification	93
5.1	Integrator(s) Design	93
5.2	Feedback DAC Design	103
5.3	3-bit Flash ADC Design	108
6	Top-level Simulation Results	115
6.1	Thermal Noise Simulations	115
6.2	Transient Simulations	115
6.3	Performance Summary and Comparison	116
	Conclusions and Future Work	121
	Acknowledgments	123
	References	125

List of Tables

2.1	Optimized NTF Zeros Positions for an N_{th} Order Modulator	19
2.2	Benefits that are obtained by using a CT and a DT loop filter	26
3.1	ADC target specifications	27
3.2	Ranges for the NTF design parameters	33
3.3	Design space solutions for best <i>OOBG-MSA</i> compromise. (<i>Bold: disadvantage, Underlined: selected NTF</i>)	39
3.4	CT equivalents for rectangular feedback DACs for the first to third-order DT low-pass loop filter poles	42
3.5	Overview of various benefits and drawbacks of various CT filter approaches	50
4.1	Component values for this work's 3^{rd} order 3 bits modulator	78
4.2	Component values for this work's 3^{rd} order 3 bits modulator	86
4.3	Sub-block specifications derived from analysis of non-idealities	91
6.1	Thermal Noise Contributions of the sub-blocks of the modulator . . .	117
6.2	Power Consumption of the modulator with individual sub-blocks contributions	119
6.3	Comparison of this work with recently published state-of-the-art. . . .	119

List of Figures

2.1	Analog to Digital Conversion: (a) Generic scheme, (b) Illustration of the signal processing (A Nyquist-rate ADC is assumed)	4
2.2	Ideal quantization process: (a) Symbolic representation, (b) Linear model of an ideal quantizer, (c) Ideal input-output curve and (d) quantization error of a 3-bit quantizer, (e) Ideal input-output curve and (f) quantization error of a 1-bit quantizer (comparator)	5
2.3	Quantization error: (a) Probability density function, (b) Power spectral density	6
2.4	Anti-aliasing filter for: (a) Nyquist-rate converters, (b) Oversampling converters. Shadowed images correspond to those created by the sampling process	8
2.5	PSD of the Quantization Error of an Oversampled Converter	9
2.6	Block Diagram of an Oversampling $\Sigma\Delta$ ADC	11
2.7	The Signal Processing in a $\Sigma\Delta$ ADC	12
2.8	The Basic $\Sigma\Delta$ Modulator Architecture and its Linear Model	13
2.9	Definitions of the Performance Metrics used to characterize a $\Sigma\Delta$ ADC	15
2.10	General Structure of a single-loop $\Sigma\Delta$ Modulator	17
2.11	Loop-Filter Common Structures	18
2.12	NTF Zeros realization in the CIFB Structure	20
2.13	NTF Zeros realization in the CIFF Structure	20
2.14	Root Locus of 2 nd and 3 rd Order $\Sigma\Delta$ Modulators	21
2.15	Peak SNR vs. oversampling ratio for different modulators topologies. SLi: i-th order single-loop modulator; Mi: i-th order 4-bit single-loop modulator; Cijk: Cascaded modulator i-j-k	23
2.16	$\Sigma\Delta$ Modulator with (a) Discrete-Time and (b) Continuous-Time Loop Filter	24
2.17	(a) Switched-Capacitor and (b) Switched-Current Feedback DAC output	25
3.1	NTF design exploration flowchart	34
3.2	Design space solutions across <i>OOBG</i> and modulator order for <i>M</i> of: a) 1bit, b) 2bits, c) 3bits and d) 4bits	35
3.3	Selected solutions from design space	37
3.4	Maximum stable amplitude (<i>MSA</i>) in the design space	38
3.5	Simulated spectrum of selected NTF. <i>f</i> _{sig} =1-MHz, <i>P</i> _{sig} ≈ <i>MSA</i> (-3.5dB)	38

3.6	Loop filter representation for DT and CT modulators (a) DT modulator; (b) CT modulator	40
3.7	NRZ and RZ feedback DAC pulse shapes with finite rise and fall times. (a) Slewing NRZ-DAC pulse; (b) slewing RZ-DAC pulse	43
3.8	Step response of DT loop filter and it's CT counterpart. Circles indicating sampling instances at quantizer input.	46
3.9	Simplified schematic of a fully differential gmC-integrator with CMFB circuit and parasitic capacitances C_p	48
3.10	Fully differential active gmC-integrator	48
3.11	Simple fully differential active RC-integrator	49
3.12	Linear model of CRFB and CRFF topology	51
3.13	Equivalent representation of a CT $\Sigma\Delta$ modulator. (a) General CT $\Sigma\Delta$ modulator architecture; (b) modified representation of a CT $\Sigma\Delta$ modulator	52
3.14	Signal transfer function of CRFB and CRFF topology showing difference in anti-aliasing and peaking	53
3.15	Focus on first alias band of CRFB and CRFF signal transfer function.	54
3.16	Passive summation network prior to quantizer input with parasitic capacitance, C_{par}	55
3.17	Proposed Active-RC architecture with capacitive feedforward compensation.	56
4.1	Linear model of CRFF with the addition of integrator nodes scaling factors, a_1, a_2, a_3	60
4.2	Scaling of a signal node. Note how x_1 is scaled by a factor of 'a' without affecting x_2 or FF_2	60
4.3	Transient simulation showing integrator outputs for a 0.1FS (left) and FS (right) sinusoidal input	61
4.4	Quantization noise extraction, E_Q , for NTF verification	61
4.5	NTF transient verification	62
4.6	Integrator output swings versus DC input level	63
4.7	SNR versus DC input level. Note: A 3-dB increase exists due to the input being DC instead of sinusoidal.	63
4.8	Quantizer input (blue) showing larger swing relative to itself at the sampling instances (red).	64
4.9	Thermal noise model of loop filter showing noise addition of each noise contributor.	66
4.10	Noise in a differential pair representing amplifier input referred noise	67
4.11	Resistive feedback DAC implementation	69
4.12	Current-steering feedback DAC architectures	70
4.13	In-band noise (IBN) versus DC gain of amplifiers	81
4.14	Effect of RC product variations on the MSA and the IBN of modulator	82
4.15	Example from [Paton 04] showing trimming done to all integration capacitors	83

4.16	Nonideal and ideal integrator step response	84
4.17	GBW effect on <i>IBN</i> of the modulator	85
4.18	DWA Sample Response for Elements Selection	87
4.19	DAC Linearity effect on SNDR with and without DWA enabled. . . .	88
4.20	Effect of jitter on SNDR of modulator	90
4.21	Effect of Quantizer Offset on modulator's SNDR	90
5.1	Schematic of the telescopic amplifier used in amplifier of first integrator	94
5.2	Figure showing necessary but not sufficient bias current needed in first amplifier output stage.	95
5.3	Basic model showing the impedances that impact the closed-loop GBW of the first integrator.	96
5.4	Closed loop gain-bandwidth product as function of overdrive voltage and bias current for the first stage amplifier	96
5.5	DC gain as function of overdrive voltage and bias current for the first stage amplifier	97
5.6	Effect of increasing differential pair ratio for fixed bias current (re- ducing overdrive voltage) on gain-bandwidth product.	97
5.7	Telescopic amplifier with final scalable sizing for the first integrator .	98
5.8	Conventional common-mode feedback method	99
5.9	New common-mode feedback method avoids loading the amplifier output with extra resistors.	100
5.10	Signal-to-Noise and Distortion ratio versus tail current (g_m) of the first integrator. The modulator is simulated with the first integrator as transistor level while the rest as behavioural.	101
5.11	Power Spectral Density of Modulator Output for two bias current levels: 160 and 350 μ A	102
5.12	Symmetrical OTA architectures implemented in the third integra- tor/adder stage	103
5.13	Power Spectral Density of the Modulator Output for no feedback delay compensation. All three amplifiers in transistor level while the rest of the modulator as behavioural.	104
5.14	Power Spectral Density of the Modulator Output for 0.25 T_s feedback delay compensation. All three amplifiers in transistor level while the rest of the modulator as behavioural.	105
5.15	Push-Pull current-steering DAC implementation	105
5.16	Monte Carlo simulations showing normalized current mismatch of DAC seven unit elements. (Top: NMOS, Bottom: PMOS)	106
5.17	DAC switching logic. The NOR/NAND gates ensure proper timing of feedback pulse to current-steering elements.	107
5.18	High-crossing and low-swing cascode switching scheme implementation.	108

5.19	DAC simulation results before and after enhanced DAC switching technique (high-crossing and low-swing cascode switching scheme). [Top: DAC differential output current waveform, Bottom: DAC current source drain-node disturbances)	109
5.20	DAC output impedance seen from the virtual ground of the first integrator. (with and without cascode switching) [Top: NMOS , Bottom : PMOS]	110
5.21	Single-ended version of a 3-bit Flash ADC. For differential operation, each comparator has four inputs, two for differential input and two for differential reference input.	111
5.22	Single-ended circuit implementation for the energy efficient two-stage comparator	111
5.23	Final Schematic of Comparator Stage. Fully-differential implementation of Energy-efficient topology with an extra output latch stage. . .	112
5.24	Waveforms describing comparator internal operation	113
5.25	Timing Diagram of Flash ADC with feedback path. The waveforms show that the quantizer output settles prior to RZ pulse is set low. .	113
6.1	Thermal Noise AC simulations showing the in-band contributions of the loop filter with the feedback DAC [Top: Noise Spectral Density , Bottom : Integrated Noise (rms)]	116
6.2	Transient waveforms of the most critical internal nodes of the modulator	117
6.3	PSD of the modulator output for all transistor level simulation ($NPs=2048$). Dotted curve represents the integrated in-band noise from dc to 2MHz.	118
6.4	Comparison between transistor-level output spectrum and NTF design equation showing good agreement.	118

Abstract

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This thesis presents low-power system and circuit design techniques of high linearity continuous time sigma delta modulator. The target application is analog to digital conversion of wide bandwidths implemented in DSM (deep sub-micron) CMOS technologies. On the system level, a sigma delta modulator with *relaxed analog requirements* is proposed. It is based on the use of multi-bit quantization and a novel feedforward loop filter architecture. This architecture facilitated the use of low gain, power efficient telescopic OTAs (operational transconductance amplifiers) in the loop filter. On the circuit level, a low power implementation of the high linearity input feedforward modulator architecture is proposed with a new clocking scheme that reduces the timing complexity in the feedback path. The implementation eliminated the need for an additional active summer stage without the penalty of signal attenuation due to passive summation. The research also shows that *digitally assisted analog processing* is an attractive low power alternative in DSM technologies, where DEM (dynamic element matching) was used to relax the matching requirements of the feedback DAC capacitors. A power efficient implementation of the DWA (data weighted averaging) algorithm is presented and designed. The aggressive circuit level sizing and analog circuit techniques allow the realization of modulator at low power levels. A top down design methodology was followed to implement the proposed continuous-time sigma delta modulator in a $0.18\mu\text{m}$ CMOS process. The modulator achieves an ENOB of 13-bit with more than 82dB SFDR over the input bandwidth of 2 MHz at an oversampling ratio of 24X, while consuming a worst case current of 0.77mA from a 1.8V supply for both the analog and the digital parts. This resulted in a FOM (figure of merit) of 45fJ/Conversion , outperforming the state of the art continuous time and discrete time sigma delta modulators. Key words: continuous-time Sigma Delta modulator, feedforward.

Chapter 1

Introduction

Analog-to-Digital Converters (ADC) are a necessity in all applications that interface with physical signals. They pose as the most challenging block in any application-specific integrated circuit. This thesis aims to implement an ADC utilizing a continuous-time $\Sigma\Delta$ modulator. Continuous-time ADCs claim to support higher bandwidths with lower power than their discrete-time counterparts.

1.1 Motivation

For the past few years, figure-of-merit values below 50fJ per conversion-level have been dominated by successive-approximation converters (*SAR*), which is attributed to the digital-friendly nature of their architectures. The motive for this thesis is to present a design flow for a continuous-time $\Sigma\Delta$ modulator with an increased focus on aggressively reducing power consumption to break the barrier of 50fJ per conversion-level.

1.2 Outline / Thesis Organization

The thesis is divided into six chapters including lists of contents, tables and figures as well as list of references.

Chapter 1 includes thesis introduction and motivation for choosing a continuous-time $\Sigma\Delta$ modulator as a low power consumption nominee. This chapter ends with the thesis outline.

In Chapter 2, an overview of $\Sigma\Delta$ Analog-to-Digital Converters (ADC) is presented. Basic concepts such as sampling and quantization are explained along with definitions for a number of metrics used to characterize performance of ADCs. $\Sigma\Delta$ ADC architecture is then presented with a discussion of the various factors that affect the performance of $\Sigma\Delta$ ADCs.

In Chapter 3, the system level design of the continuous-time $\Sigma\Delta$ ADC is presented. The target specifications are defined. A systematic approach is followed to select