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شبكة المعلومات الجامعية

التوثيق الالكتروني والميكروفيلم

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CAIRO UNIVERSITY
INSTITUTE OF STATISTICAL STUDIES AND
RESEARCH, CAIRO UNIVERSITY
EGYPT

ON GOODNESS-OF-FIT TESTS BASED ON EMPIRICAL CHARACTERISTIC FUNCTION

By

Suzanne Abdel-Rahman Sayyid Allam

Under the Supervision of

Professor Samir Kamel Ashour
Professor of Mathematical
Statistics-Institute of Statistical Studies and
Research-Cairo University

Professor Ahmed Fouad Mohamed
Professor of Mathematical
Statistics-Institute of Statistical Studies and
Research-Cairo University

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Suzanne Abdel-Rahman Sayyid Allam

This Thesis for the PhD In Mathematical Statistics , Instiute Of Statistical Studies and Research ,Cairo University , and has been approved by

Name	Signature
....	....
...M. E. Habil...	....M. E. Habil...
...Ahmed....Fouad	...Ahmed...Fouad
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Table of Contents

Page

Page

ABSTRACT

Characteristic functions (CF) were originally developed as a tool for the solution of problems in probability theory and admit many important applications in this branch of Mathematics as well as in Mathematical Statistics. The empirical characteristic function (ECF) is the sample counterpart of the CF. It was defined by Parzen (1962) and can be used in statistical inference. It can be used for parameter estimation and hypothesis testing.

In the literature, there are many studies which introduced goodness-of-fit tests based on the ECF using different methodologies. The basic idea of the ECF method is to compare the CF derived from the hypothesized model with the ECF obtained from the sample data.

In this thesis, goodness-of-fit tests based on the ECF are used for testing the fit of the generalized exponential distribution. In addition, a power comparison is conducted with other common goodness-of-fit tests that are the tests based on empirical distribution function (EDF). Also, the effect of the estimation method used for estimating the unknown parameters of the generalized exponential distribution on the power of the ECF test is studied. Finally, the sampling distribution for the ECF test statistic is obtained using Pearson system.

Chapter I

INTRODUCTION

A statistical problem encountered in many areas of research is the need to assess whether a sample of observations comes from a specified distribution. Typically such situations are known as 'Goodness of Fit' problems, that is, how well are the data modeled by a certain distribution? A goodness-of-fit test uses the properties of a hypothesized distribution to assess whether a sample of observations is generated from that distribution.

Characteristic functions (CF) were originally developed as a tool for the solution of problems in probability theory and admit many important applications in this branch of Mathematics as well as in Mathematical Statistics. The empirical characteristic function (ECF) is the sample counterpart of the CF. It was defined by Parzen (1962) and can be used in statistical inference. It can be used for parameter estimation and hypothesis testing.

In the literature, there are many studies which introduced goodness-of-fit tests based on the ECF using different methodologies. The basic idea of the ECF method is to compare the CF derived from the hypothesized model with the ECF obtained from the sample data.

The generalized exponential distribution was introduced by Gupta and Kundu (1999). This distribution can be used quite effectively in analyzing many lifetime data, particularly in place of the two-parameter gamma and Weibull distributions. It is observed that the generalized exponential distribution can be considered for situations where a skewed distribution for a non-negative random variable is needed.

In this thesis, goodness-of-fit tests based on the ECF are used for testing the fit of the generalized exponential distribution. In addition, a power comparison is conducted with other common goodness-of-fit tests that are the tests based on empirical distribution function (EDF). Also, the effect of the estimation method used for estimating the unknown parameters of the generalized exponential distribution on the power of the ECF test is studied. Finally, the sampling distribution for the ECF test statistic is obtained using Pearson system. All the Algorithms in this thesis are implemented using the Mathcad (version 13) software.

This thesis is organized as follows. Some important definitions and notation are introduced in Chapter II. Chapter III includes a literature review about different goodness-of-fit tests that are based on ECF. Chapter IV, which is the main analytical chapter, introduces a power comparison between the ECF and EDF tests, for testing the fit of the generalized exponential distribution. In addition, the same chapter includes a study about the effect of parameter estimation method on the power of the ECF test for testing the fit of the generalized exponential distribution. Finally, Chapter IV contains a section about obtaining the sampling distribution of the ECF test statistic using Pearson system.

The results of all simulation experiment conducted in this thesis are organized in tables in appendix A. Also, appendix B contains all the Mathcad programs implemented in this thesis.

Chapter II

DEFINITIONS AND NOTATION

This chapter is devoted to some important definitions and notation that will be used in the present dissertation.

(2.1) Methods of Estimation

In the literature, there are different methods for estimating the unknown parameters of a statistical distribution. The most commonly used among these methods are the method of moments and method of maximum likelihood.

(2.1.1) Method of Moments

The method of moments (MM) is a technique for constructing estimators for the parameters which is based on matching the sample moments with the corresponding population moments. It is frequently called the method of moments because it is understood that, whenever possible, the parameter should be estimated by using moments, particularly, the lowest order moments that are convenient. The MM method provides estimators that are consistent but not as efficient as the maximum likelihood ones. It is often used because it leads to very simple computations, unlike maximum likelihood method which can become very cumbersome.

The MM method consists of equating the first few moments of a population to the corresponding moments of a sample, thus getting a number of equations that are needed to be solved in terms of the unknown parameters of the population. Therefore, if a population has k unknown parameters $\theta_1, \theta_2, \ldots, \theta_k$, then the parameter estimates $\overline{\theta}_1, \overline{\theta}_2, \ldots, \overline{\theta}_k$ can be obtained by solving the following system of simultaneous equations: