

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



# شبكة المعلومات الجامعية التوثيق الالكتروني والميكرو فيلم



شبكة المعلومات الجامعية

# جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

## قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها  
على هذه الأفلام قد أعدت دون أية تغيرات



## يجب أن

تحفظ هذه الأفلام بعيدا عن الغبار

في درجة حرارة من ١٥-٢٥ مئوية ورطوبة نسبية من ٢٠-٤٠%

To be Kept away from Dust in Dry Cool place of  
15-25- c and relative humidity 20-40%

# بعض الوثائق الأصلية تالفة

# بالرسالة صفحات لم ترد بالاصل

# **PATCHING METHODS IN FLUID MECHANICS**



THESIS  
SUBMITTED FOR PH.D. DEGREE IN SCIENCE  
(APPLIED MATHEMATICS)

BY  
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To  
FACULTY OF SCIENCE  
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**1996**

*Handwritten signature*



سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنْتَ  
الْعَلِيمُ الْحَكِيمُ

صدق الله العظيم

سورة البقرة (٣٢)

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**TO MY PARENTS,**

**MY WIFE ABIR AND OUR CHILDREN**

**MOHAMED, AHMED, AND MOATAZ**

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**Elsayed Mabrouk Abd El-Baky**

## **SUMMARY**

**The thesis comprises six chapters. In the thesis a new method has been deduced to solve Navier- Stokes equations for compressible and incompressible fluid flows the results have been compared to those resulted by using Beam and Warming techniques.**

**In the first chapter exhibits most of the known methods in question such as**

**1- The Non- Primitive variable Approach**

**(i)The Velocity- Vorticity Approach**

**(ii)The Vorticity- Stream function Approach**

**2- The Primitive variable Approach**

**(a) The Artificial Compressibility Approach**

**(b) The Primitive- Variable Approach with Pressure Poisson Equation**

**(c) Compatibility Condition**

**(d) The present Approach**

**In the second chapter the Navier- Stokes equations are derived together with the continuity equation, momentum equation and energy equation.**

**In the third chapter the author shows the way of deriving a new method in two-dimensional and in three- dimensional(Cartesian and generalized curvilinear coordinate systems) , in the following cases:-**

## **1- Incompressible fluid flow**

## **2- Compressible fluid flow**

The fourth chapter deals with the accuracy and stability of a new scheme applied to the heat equation(diffusion equation) as an example.

In the fifth chapter the new scheme is applied in a fractional step method. In the last chapter the author introduces applications to incompressible and compressible fluids.

In the incompressible cases:-

(I) The driven cavity in two-dimensional

(II) The driven cavity in three- dimensional

Nevertheless, for the compressible flow the Navier- Stokes equations are solved for the case of flow around a wedge. This new method is also very useful in it presents parallel computing which is in itself a time saving procedure applicable on future parallel computing machines.

It is worthwhile noting that the programs throughout the thesis have been written in FORTRAN 77 language , FORTRAN 90 and run on UNIX system supercomputer[CRAY Y- MP, CRAY X- MP and T-3D] at Columbus and the computer of Aerospace department, faculty of Engineering , University of Cincinnati, Cincinnati, OH, USA 45221-70.

## List of Symbols

$x, y, z$  : Cartesian Coordinates.

$\xi, \eta, \zeta$  : Generalized Curvilinear Coordinates.

$t$  : Time parameter

$V$  : Velocity Vector.

$u$  :  $x$ - component of Velocity.

$v$  :  $y$ - component of Velocity.

$w$  :  $z$ - component of Velocity.

$p$  : Static Pressure divided by density.

$\rho$  : Density.

$P$  : Thermodynamic Pressure.

$E_t$  : Total Energy per Unit Mass.

$e$  : Internal Energy per Unit Mass.

$\tau_{ij}$  : Stress Tensor.

$q$  : Heat Transfer,

$\mu$  : Coefficient of Viscosity.

$k$  : Coefficient of Conductivity.

$Q$  : Conserved Variables.

$E, F, G$  : Flux Vector.

$\gamma$  : Ratio of Specific Heats.

$C_p$  : Specific Heat at Constant Pressure.

$R$  : Gas Constant.

$Re$  : Reynolds Number.

$Pr$  : Prandtl Number.

$\tau$  : Computational Time.

$\Delta$  : Change in Variables.

$\partial$  : Partial of Variables.

**LHS : Left Hand Side.**

**RHS : Right Hand Side.**

$$\Delta Q^n : Q^{n+1} - Q^n$$

$\infty$  : Free Stream Value.

**A, B, C : Flux Jacobians.**

**O : Higher Order Terms.**

**I : Identity Matrix.**

**L : Discrete Laplacian Operator.**

**D : Discrete Divergence Operator.**

**G : Discrete Gradient Operator.**

**H : Discrete Convective Operator.**

$\delta$  : Discrete Space Derivative Operator.

$\Delta t$  : Time increment.

$\Delta x$  : Space increment in x- direction.

$\Delta y$  : Space increment in y- direction.

### **Superscripts**

**n, n+1 : Time level at t and t+ $\Delta t$ .**

**\*, \*\* : Intermediate time levels.**

### **Subscripts**

**i,j : Grid indicis in x- direction and y- direction, respectively.**

**x,y : First order derivatives with respect to x- and y- directions, respectively.**

**xx,yy : Second order derivatives with respect to x- and y- directions, respectively.**