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Generalization of Some Statistical Distributions Using Kumaraswamy Distribution

A Thesis

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SUMMARY

The modeling and analysis of lifetime is an important aspect of statistical work in a wide variety of scientific and technological fields. The failure behavior of any system can be considered as a random variable due to the variations from one system to another resulting from the nature of the system. Therefore, it seems logical to find a statistical model for the failure of the system. In other applications, survival data are categorized by their hazard rate, For example, there are distributions with fixed hazard rate such as the exponential distribution. Other distributions are characterized by incremental hazard rate. Some have decreasing failure rate, and others combine the three kinds on different time periods to appear in the the form of bathtub.

The Kumaraswamy distribution is similar in its simplest form to the beta distribution in terms of probability density function and cumulative distribution function. This distribution has an advantage to the beta distribution, because it is simpler to use especially in simulation studies.

The Kumaraswamy distribution is applicable to many natural phenomena , such as the heights of individuals, scores obtained on a test, atmospheric temperatures, hydrological data and landslides. It can be a useful tool to analyze customer lifetime duration in marketing research and can be used quite effectively in analyzing real data. This distribution is in use in electrical, civil, mechanical, and financial engineering applications.

The Kumaraswamy distribution arises depending on order statistics, and its form clearly does not depend on special functions, thus have a distinct role in ease of statistical modeling, it also has a tendency for application in educational uses.

In recent years, generalized distributions have been widely studied in statistics as they possess flexibility in applications. This is justified because the traditional distributions often do not provide good fit in relation to real data set studied.

In this thesis, according to Kumaraswamy distribution we propose a new class of generalized distributions called the Exponentialed Kumaraswmay Lindley (EKumL) that is capable of modeling bathtub-shaped hazard function. The beauty and importance of this distribution lies in its ability to model monotone and non-monotone failure rate function, which are quite common in lifetime data analysis and reliability.

The thesis consists of three chapters as follows:

Chapter I: contains the concepts and basic characteristics of the Lindley distribution and its applications. Some of the previous research on the different forms of the Lindley distribution are presented.

Chapter II: we introduce Kumaraswamy distribution and we present some statistical properties such as the mode, quantile function and moments. In addition, estimation of the parameters using the maximum likelihood method and display elements of the information matrix. We present many of distributions using generalization Kumaraswamy distribution and the analytical shapes of the corresponding probability density functions are derived with graphical illustration. Applications using real data sets are given.

Chapter III: constitutes our main goal, which is a complete review of the Exponential Kumaraswamy Lindley distribution. Some basic properties of this distribution, such as quantil function, moments, moment generating function and entropy are derived. as well as the derivation of maximum likelihood estimates of the parameters and the observed and expected information matrix. Finally, numerical examples are given using sets of real data, A simulation study is conducted to demonstrate the effect of the sample on the estimates of the parameters and its characteristics. The results of this chapter are published in "Journal of Statistics: Advances in Theory and Applications". 2015, 14(1), 69-105.

CHAPTER(I)
LINDLEY DISTRIBUTION AND ITS APPLICATION

1.1 Introduction

In many applied sciences such as medicine, engineering and finance, amongst others, modeling and analyzing lifetime data are crucial. Several lifetime distribution have been used to model such kinds of data. For instance, the exponential, Weibull, gamma, Rayleigh distributions and their generalizations [see, e.g., Gupta and Kundu, (1999)]. Each distribution has its own characteristics due specifically to the shape of the failure rate function which may be only monotonically decreasing or increasing or constant in its behavior, as well as non-monotone, being bathtub shaped or even unimodal. The quality of the procedures used in a statistical analysis depends heavily on the assumed probability model or distribution. Because of this, considerable effort has been expended in the development of large classes of well-known probability distributions along with relevant statistical methodologies. However, there still remain many important problems where the real data does not follow any of the classical probability models. The Lindley distribution was introduced by Lindley (1958) as a new distribution useful to analyze lifetime data especially in applications modeling stress-strength reliability. Besides, some researchers have proposed new classes of distributions based on modifications of the Lindley distribution, including also their properties. The main idea is always directed by embedding former distributions to more flexible structures. In a recent paper Ghitany et al (2008c) studied the properties of the Lindley distribution under a carefully mathematical treatment. Showing that this distribution may provide a better fit than exponential distribution based on the waiting time at a bank for service. The use of the Lindley distribution could be a good alternative to analyze lifetime data within the competing risks approach as compared with the use of negative Exponential or even the Weibull distribution commonly used in this area. The exponential distribution assumes constant hazard function, and hence is usually not an appropriate assumption for many competing risks.

Lindley (1958), introduced a one-parameter distribution, known as Lindley distribution. Its probability density function (pdf) is given by

$$f(x) = \frac{\theta^2}{\theta + 1}(1 + x)e^{-\theta x} \quad , x > 0, \theta > 0 \tag{1.1}$$

The cumulative distribution function (cdf) of Lindley distribution is obtained as

$$F(x) = 1 - e^{-\theta x}\left[1 + \frac{\theta x}{\theta + 1}\right], x > 0, \theta > 0 \tag{1.2}$$

A distribution that is close in form (1.1) is the well-known exponential distribution given by the pdf

$$f(x) = \theta e^{-\theta x}, x > 0, \theta > 0.$$

However, due to the popularity of the exponential distribution in statistics and many applied areas, the Lindley distribution given by (1.1) has been overlooked in the literature. The aim of this chapter is provide a comprehensive description of its mathematical properties. We show that in many applications the Lindley distribution is a better model than the exponential distribution.

The rest of this chapter is organized as follows. In the next section we introduce various statistical properties of the Lindley distribution including the mode, moments, hazard rate function, cumulants and Entropy. We present maximum likelihood estimates of the parameter using real data application of the Lindley distribution are given in Section (1.3) and (1.4). A simulation study is conducted in Section (1.5). Finally, in Section (1.6) we present some of the various generalizations of the Lindley distribution presenting the probability density functions with their graphs for different values of the parameters.

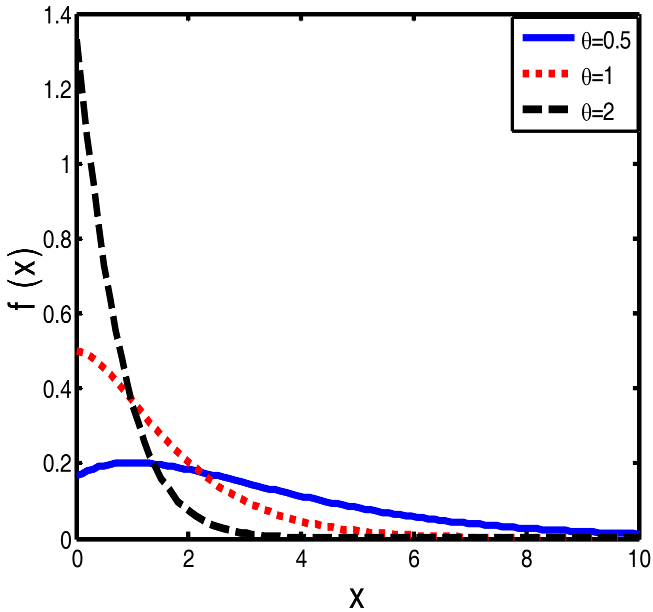

Figure (1.1). The pdf of the Lindley distribution for different values of the parameters.

1.2 Some properties of the Lindley Distribution

In this section, we present some of the basic statistical properties of the Lindley distribution, in particular, the mode, moments and related measures including coefficients of variation, skewness and kurtosis. Also, the characteristic function and cumulant generating function are presented.

1.2.1 Mode of the Lindley distribution

From Equation (1.1) it follows that

$$\frac{d}{dx}f(x) = \frac{\theta^2}{\theta+1}(1-\theta-\theta x)e^{-\theta x}.$$

The nature of Lindley distribution for different values of its parameter θ has been shown graphically in Figure (1.1). The density function of the Lindley distribution can take different shapes. For $\theta < 1$, $(d/dx)f(x) = 0$ implies that $x_0 = \left((1-\theta)/\theta\right)$ is the unique critical point at which $f(x)$ is maximized. For $\theta \geq 1$, $(d/dx)f(x) \leq 0$, i.e. $f(x)$ is decreasing in x.