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PROBLEMS OF GEOMETRIC FIELD THEORIES

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To My Loving Parents

and

My beautiful daughter

♡ *Mariam* ♡

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List of Abbreviations

AP	Absolute Parallelism
BB	Building Blocks
CMBR	Cosmic Microwave Background Radiation
Ein-Max	Einstein-Maxwell
ENG	Einstein Non Symmetric Geometry
ENT	Einstein Non Symmetric Theory
EM	Electromagnetism
GFT	Generalized Field Theory
GR	General Relativity
LHS	Lift Hand Side
M	Manifold
MT	Møller Theory
NGR	New General Relativity
PAP	Parameterized Absolute Parallelism
Phen.	Phenomenological
RC	Riemann-Cartan
RHS	Right Hand Side
Riem.	Riemannian
SG	Symmetry Group
SN	Super Nova

SR	Special Relativity
TEGR	Teleparallel Equivalent of General Relativity
n	Dimension of the space
N	Natural number
()	Used for symmetrization
[]	Used for anti-symmetrization