

AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
Engineering Physics and Mathematics

Plate Stresses and Deformations Due to In-Plane and Out-Plane Loads

A Thesis submitted in partial fulfilment of the requirements of the
degree of

Master of Science in Physics, Engineering Mathematics and
Engineering Mechanics
(Engineering Physics and Mathematics)

By

Eslam Nabil Shawki El-Ganzoury

Demonstrator of Engineering Mechanics
(Engineering Physics and Mathematics)
Faculty of Engineering, Ain Shams University

Supervised By

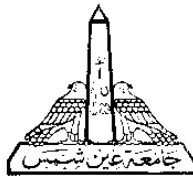
Prof. Dr. Abd-Allah Mostafa El-Marhomy

Department of Engineering Physics and Mathematics,
Faculty of Engineering, Ain Shams University

Associate Prof. Said Yousif Aboul-Haggag

Structural Engineering Department, Faculty of Engineering,
Ain Shams University

Cairo, Egypt - (2016)



AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING

Engineering Physics and Mathematics

**Plate Stresses and Deformations
Due to In-Plane and Out-Plane Loads**

A Thesis submitted in partial fulfilment of the requirements of the
degree of

Master of Science in Physics, Engineering Mathematics and
Engineering Mechanics

(Engineering Physics and Mathematics)

By

Eslam Nabil Shawki El-Ganzoury

Postgraduate Diploma in Physics, Engineering Mathematics
and Engineering Mechanics

(Engineering Physics and Mathematics)

Faculty of Engineering, Ain Shams University, 2013

Supervised By

Prof. Dr. Abd-Allah Mostafa El-Marhomy

Department of Engineering Physics and Mathematics,
Faculty of Engineering, Ain Shams University

Associate Prof. Said Yousif Aboul-Haggag

Structural Engineering Department, Faculty of Engineering,
Ain Shams University

Cairo, Egypt - (2016)

Researcher Data

Name: Eslam Nabil Shawki El-Ganzoury

Date of birth: 3 February 1988

Place of birth: Cairo, Egypt

Last academic degree: Postgraduate Diploma in Physics, Engineering Mathematics and Engineering Mechanics

Field of specialization: Engineering Mechanics

University issued the degree: Ain shams University

Date of issued degree: 2013

Current job: Demonstrator of Engineering Mechanics in Department of Engineering Mathematics and Physics, Faculty of Engineering, Ain Shams University

Thesis Summary

The present thesis introduces new displacement and stress functions for rectangular plate in general coordinates under the entire applicable boundary conditions with any in plane or out-plane loading. These developed functions are easy and fast to be used by engineers in construction sites or manufacturing facilities by just simple calculator or Excel ® sheets. The method of analysis depends mainly on the minimum energy concept and appropriate real polynomials in the functions at the points coordinates over the plate area. The current study introduces then a simple method of analysis seeking acceptable and accurate results. The bending moment and shear forces functions can also be derived from the present achieved displacement functions.

The results from the present derived equations are compared to the results of Timoshenko of the same plate cases if possible, and to new analytical methods like symplectic method for the cases not presented by Timoshenko. Moreover, Ansys ®, the design program, is also used to compare the present results for some cases. Many different assumptions and trials were carried on to most cases of plate problems to seek acceptable and accurate results, which are characterized by easiness and fastness in use.

The calculations and the derived functions were carried on by the aid of the well-known mathematical programs “Matlab ® and Maple ®”. They were used in finding derivatives and integrations of the mathematical functions.

This thesis consists of nine chapters and an appendix including a conclusion part and list of figures, list of tables, list of symbols, and bibliography:

Chapter 1: This introductory chapter shows the motivation and the target of this work. It focuses on the scope of this thesis and clarifies its organization flow.

Chapter 2: This chapter contains most required information for the stresses and deflection of the plates and mathematical history of the study of their problems. It also includes definitions of the subject and important terms.

Chapter 3: This chapter presents information on the deformation of the rectangular plates $a \times b$ with no free edges. It contains a conclusion about the results from the new derived equations for the plate deformation with the compared results.

Chapter 4: This chapter presents information on the deformation of the rectangular plates $a \times b$ with two opposite free edges and others are not free. It contains a conclusion about the results from the new derived equations for the plate deformation with the compared results.

Chapter 5: This chapter presents information on the deformation of the rectangular plates $a \times b$ with only one free edge and others are not free. It contains a conclusion about the results from the new derived equations for plate the deformation with the compared results.

Chapter 6: This chapter presents information on the deformation of the rectangular plates $a \times b$ with two adjacent free edges and others are not free. It contains a conclusion about the results from the new derived equations for the plate deformation with the compared results.

Chapter 7: This chapter presents information on the deformation of the rectangular plates $a \times b$ with three free edges and the other is not free. It contains a conclusion about the results from the new derived equations for the plate deformation with the compared results.

Chapter 8: This chapter presents information on the deformation of the rectangular plates $2a \times 2b$ with all edges free. It contains a conclusion about the results from the new derived equations for the plate deformation with the compared results.

Chapter 9: This chapter presents information on the deformation of the circular plates $r = a$ with different boundary conditions. It contains a conclusion about the results from the new derived equations for the plate deformation with the compared results.

Key words: plate, deformation, stresses, simply supported edge, clamped edge, free edge, minimum energy concept, in-plane load, out-plane load

ACKNOWLEDGEMENTS

First and above all, I praise *Allah*, the almighty for providing me with strength and patient to write this thesis and whose many blessings guided me. Peace is upon the prophet, his companions, and all who followed him until the Day of Judgment.

I like to express my appreciation to **Prof. Dr. Abd-Allah Mostafa El-Marhomy** for his invaluable assistance and support. His knowledge and wisdom was my light to advance in my thesis.

I am sincerely grateful to **Associate Prof. Said Yousif Aboul-Haggag** for his continuous guidance and encouragement throughout this thesis. His effort and help was my priceless aid in my thesis.

Finally, I must express my very profound gratitude to my parents and my brother for providing me with unfailing support and continuous encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them.

June 2016

The Table of Contents

	Page
Thesis Summary	i
Acknowledgements	iv
Table of Contents	v
List of Figures	x
List of Tables	xii
List of Symbols	xv
 Chapter (1) Introduction	 1
1.1 Background	1
1.2 The plate boundary conditions	3
1.3 The constraints of boundary conditions	6
1.4 The used comparing methods to the present results	8
1.5 Used commercial computer programs	8
1.6 The objectives of present research	8
 Chapter (2) Review of literature	 9
2.1 The definition and classification of Plates	9
2.1.1 The three groups classifying the plates	9
2.1.2 The Thin-plates and shells application s.....	10
2.1.3 The Thin-plates definition	10
2.1.4 The in-plane and out-plane definitions	10
2.2 History of researches and studies on thin plates deformation and bending	11
2.2.1 The first mathematical historical trials of plate problems.....	11
2.2.2 The reason for needing approximate solution instead of analytical solution	13
2.2.3 A quick discuss about each approximate solution method.....	14
2.3 Deformation	15
2.3.1 The deformation definition	15
2.3.2 The deformation types	15
2.4 Solving plate problems in elasticity	15
2.5 Stress definition and types	16

2.6 Quick preview on the Minimum potential energy method	16
2.7 Quick historical review and definition of symplectic method as one of main comparison methods in present thesis.....	18
Chapter (3) Rectangular plate with no free edges	20
3.1 Case (1) Fully simply supported rectangular plate $a \times b$ (SSSS)	20
3.1.1 First assumption (no. 1) for setting the $f(x)$ and $g(y)$ function.....	20
3.1.2 Conclusion of part (3.1.1)	25
3.1.3 Second assumption (no. 2) of increasing the terms x^6 and y^6 in the assumed functions $f(x)$ and $g(y)$ respectively.....	28
3.1.4 Conclusion of part (3.1.3)	33
3.1.5 Third assumption (no. 3) for setting the $f(x)$ and $g(y)$ functions...	33
3.1.6 Conclusion of part (3.1.5)	35
3.2 Case (2) Fully clamped rectangular plate $a \times b$ (CCCC)	43
3.2.1 First assumption (no. 1) for setting the $f(x)$ and $g(y)$ functions.....	43
3.2.2 Conclusion of part (3.2.1)	45
3.2.3 Second assumption (no. 2) for setting the $f(x)$ and $g(y)$ functions	46
3.2.4 Conclusion of part (3.2.3)	48
3.2.5 Comparing results with ANSYS ® program	48
3.3 Case (3) Two opposite edges are simply supported and other edges are clamped rectangular plate $a \times b$ (SSCC)	53
3.3.1 Case of uniform distributed load q over the whole area	53
3.3.2 Conclusion of part (3.3.1)	54
3.3.3 Case of a triangle load with a maximum intensity of q at $x = a$ over the whole area	55
3.3.4 Conclusion of part (3.3.3)	56
3.4 Case (4) Three edges are simply supported and one edge is clamped rectangular plate $a \times b$ (SSCS)	60
3.5 Case (5) Three edges are clamped and one edge is simply supported rectangular plate $a \times b$ (CCCS)	65
3.6 Case (6) Two adjacent edges are clamped and other edges are simply supported rectangular plate $a \times b$ (CSCS)	70
Chapter (4) Rectangular plates with two opposite free edges	75
4.1 Case (7) Two opposite free edges and other edges are simply supported	

for rectangular plate $a \times b$ (SSFF)	75
4.1.1 The plate under uniformly distributed load q over whole area	77
4.1.2 Conclusion of part (4.1.1)	78
4.1.3 The plate under uniformly distributed load q over whole area and combined with in-plane uniform load N	79
4.2 Case (8) Two opposite edges are clamped and other edges free rectangular plate $a \times b$ (CCFF)	84
4.2.1 The plate under uniformly distributed load q over whole area	86
4.2.2 Conclusion of part (4.2.1)	87
4.2.3 The plate under uniformly distributed load q over whole area and combined with in-plane uniform load N	88
4.2.4 Engineering application on the plate (CCFF) loaded by uniformly distributed load (part (4.2.1))	89
4.2.5 Comparing results with ANSYS ® program	90
4.3 Case (9) Two opposite edges are free, one edge is clamped and the other edge is simply supported rectangular plate $a \times b$ (CSFF)	94
Chapter (5) Rectangular plates with single free edge	101
5.1 Case (10) Three edges are simply supported and one is free rectangular plate $a \times b$ (SSSF)	101
5.1.1 First assumption (no. 1) for setting the $f(x)$ and $g(y)$ functions ...	101
5.1.2 Conclusion of part (5.1.1)	104
5.1.3 Second assumption (no. 2) for setting $g(y)$ function	105
5.1.4 Conclusion of part (5.1.3)	108
5.2 Case (11) Three edges are clamped and one is free rectangular plate $a \times b$ (CCCF)	111
5.2.1 First assumption (no. 1) for setting the $f(x)$ and $g(y)$ function	111
5.2.2 Conclusion of part (5.2.1)	115
5.2.3 Second assumption (no. 2) for setting $g(y)$ function	115
5.2.4 Conclusion of part (5.2.3)	117
5.2.5 Comparing results with ANSYS ® program	117
5.3 Case (12) Two opposite edges are simply supported, one edge is clamped and other edge is free rectangular plate $a \times b$ (SSCF)	122
5.3.1 First assumption (no. 1) for setting the $f(x)$ and $g(y)$ functions ...	122
5.3.2 Conclusion of part (5.3.1)	125

5.3.3 Second assumption (no. 2) for setting $g(y)$ function	126
5.3.4 Conclusion of part (5.3.3)	129
5.3.5 Third assumption (no. 3) for setting $g(y)$ function	130
5.3.6 Conclusion of part (5.3.5)	131
5.4 Case (13) Two opposite edges are clamped, one edge is simply supported and the other edge is free rectangular plate $a \times b$ (CCSF)	135
5.4.1 First assumption (no. 1) for setting the $f(x)$ and $g(y)$ functions ...	135
5.4.2 Conclusion of part (5.4.1)	138
5.4.3 Second assumption (no. 2) for setting $g(y)$ function	139
5.4.4 Conclusion of part (5.4.3)	142
5.5 Case (14) Two adjacent edges are simply supported, one edge is clamped and the other edge is free rectangular plate $a \times b$ (CSSF)	146
5.6 Case (15) Two adjacent edges are clamped, one edge is simply supported and other edge is free rectangular plate $a \times b$ (CSCF)	152
Chapter (6) Rectangular plates with two adjacent free edges	159
6.1 Case (16) Two adjacent edges are simply supported and other edges are free rectangular plate $a \times b$ (SFSF)	159
6.1.1 First assumption (no. 1) for setting the $f(x)$ and $g(y)$ function	159
6.1.2 Conclusion of part (6.1.1)	161
6.1.3 Second assumption (no. 2) for setting $g(y)$ function	161
6.1.4 Conclusion of part (6.1.3)	165
6.2 Case (17) Two adjacent edges are clamped and other edges are free rectangular plate $a \times b$ (CFCF)	167
6.2.1 First assumption (no. 1) for setting the $f(x)$ and $g(y)$ functions ...	167
6.2.2 Conclusion of part (6.2.1)	169
6.2.3 Second assumption (no. 2) for setting $g(y)$ function	169
6.2.4 Conclusion of part (6.2.3)	171
6.2.5 Third assumption (no. 3) for setting $g(y)$ function	172
6.2.6 Conclusion of part (6.2.5)	173
6.2.7 Fifth assumption (no. 3) for setting $g(y)$ function	173
6.2.8 Conclusion of part (6.2.7)	176
6.3 Case (18) Two adjacent edges are free, one edge is simply supported and other edge is clamped rectangular plate $a \times b$ (CFSF)	178

Chapter (7) Rectangular plate with three free edges	182
7.1 Case (19) One edge is simply supported edge and other edges are free rectangular plate $a \times b$ (SFFF)	182
7.2 Case (20) One edge is clamped and other edges are free rectangular plate $a \times b$ (CFFF)	184
7.2.1 Conclusion	187
7.2.2 Comparing results with ANSYS ® program	188
Chapter (8) Rectangular plate with all edges are free and supported at the corners	192
Case (21) All edges are free and supported at the corners rectangular plate $a \times b$ (FFFF)	192
Chapter (9) Circular plates	198
9.1 Case (a) Fully clamped uniform centered circular plates	198
9.1.1 First assumption (no. 1) of setting w function	198
9.1.2 Second assumption (no. 2) of setting w function	200
9.2 Case (b) Fully simply supported circular plate.....	201
Conclusion and Recommendation	205
Appendix	208

List of Figures

Figure	Title of Figures	Page
(1.1)	All the possible boundary conditions notifications S, C and F for the rectangular plate and the circular plate edges	3
(1.2)	The rectangular plate ($a \times b$) coordinates for the cases (1 to 20) of boundary conditions	5
(1.3)	The rectangular plate ($2a \times 2b$) coordinates for the case (21) of boundary conditions	5
(1.4)	The uniformly centered circular plate coordinates	6
(1.5)	The curvilinear boundary of the plate	6
(2.1)	The length L and the thickness t of a plate and the associated (x, y, z) coordinate system as t along the z direction	9
(2.2)	The midplane of the rectangular plate	11
(2.3)	Different stress measures	16
(3.1)	Fully simply supported rectangular plate $a \times b$ (SSSS)	20
(3.2 a)	Fully clamped rectangular plate $a \times b$ (CCCC)	43
(3.2 b)	Fully clamped rectangular plate 20×10 (CCCC) with Ansys	49
(3.3)	Two opposite edges are simply Supported other edges are clamped rectangular plate $a \times b$ (SSCC)	53
(3.4)	Three edges are simply supported and one edge is clamped rectangular plate $a \times b$ (SSCS)	60
(3.5)	Three edges are clamped and one edge is simply supported rectangular plate $a \times b$ (CCCS)	65
(3.6)	Two adjacent edges are clamped and other edges are simply supported rectangular plate $a \times b$ (CSCS)	70
(4.1)	Two opposite free edges and other edges are simply supported for rectangular plate $a \times b$ (SSFF)	75
(4.2 a)	Two opposite edges are clamped and other edges free rectangular plate $a \times b$ (CCFF)	84
(4.2 b)	An engineering application on the case of (CCFF) plate	89
(4.2 c)	Two opposite edges are clamped and other edges free rectangular plate 20×10 (CCFF) with Ansys	90
(4.3)	Two opposite edges are free, one edge is clamped and the other edge is simply supported rectangular plate $a \times b$ (CSFF)	94
(5.1)	Three edges are simply supported and one is free rectangular plate $a \times b$ (SSSF)	101