

شبكة المعلومات الجامعية







شبكة المعلومات الجامعية التوثيق الالكتروني والميكروفيلم



شبكة المعلومات الجامعية

### جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

### قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها على هذه الأفلام قد أعدت دون أية تغيرات



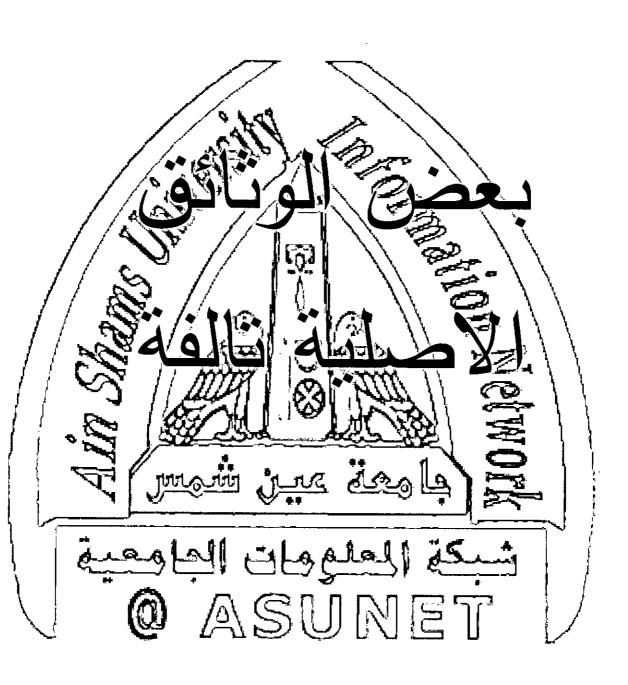
يجب أن

تحفظ هذه الأفلام بعيدا عن الغبار هي درجة حرارة من ١٥-١٥ مئوية ورطوية نسبية من ٢٠-٤٠ في درجة حرارة من ١٥-١٥ مئوية ورطوية نسبية من ٢٠-١٠ ثالثانية من ٢٥-١٥ الله عند المعادة ال



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## FINITE-DIFFERENCE METHODS AND SIMILARITY SOLUTIONS FOR SOLVING SOME PROBLEMS IN FLUID DYNAMICS

A Thesis
Submitted to The Faculty of Science
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(Mathematics)

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# Nomenclature

### Nomenclature

A	B.	C'	S.	m,	n.	r.
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 $a_1, a_2$ 

 $T_0, U_0, K's, \alpha, \beta, \gamma$  Real constants.

Essential parameters.

Pr Prandtl number.

Re Reynolds number.

x Distance along the surface.

y Distance normal to the surface.

t Time.

u Velocity in x-direction.

v Velocity in y-direction.

Temperature.

U Potential velocity.

 $U^*$  Reference velocity.

F Dimensionless velocity.

E Total specific energy

q Heat flux vector

G Dimensionless microrotation.

L Differential operator.

f Body force

V Control volume

S Orientable surface

P Pressure

### Greek symbols

η Dimensionless coordinate.

O Dimensionless temperature.

 $\Psi$  Stream function.

μ Dynamic viscosity coefficient.

V Kinematic viscosity coefficient.

2 Second coefficient of viscosity

 $\rho$  Density of the fluid.

 $\sigma$  Stress tensor

ε Singular perturbation parameter.

#### Subscripts

w Surface Conditions.

 $\infty$  Conditions far a way form the surface.

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