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ثبكة المعلومات الجامعية







Ain Shams University
Faculty of Science
Mathematics Department

On the Efficiency and Accuracy of Numerical Solutions of Particular Differential Equations

Thesis

Submitted for the Degree of Doctor of Philosophy for Science in Pure Mathematics (Numerical Analysis)

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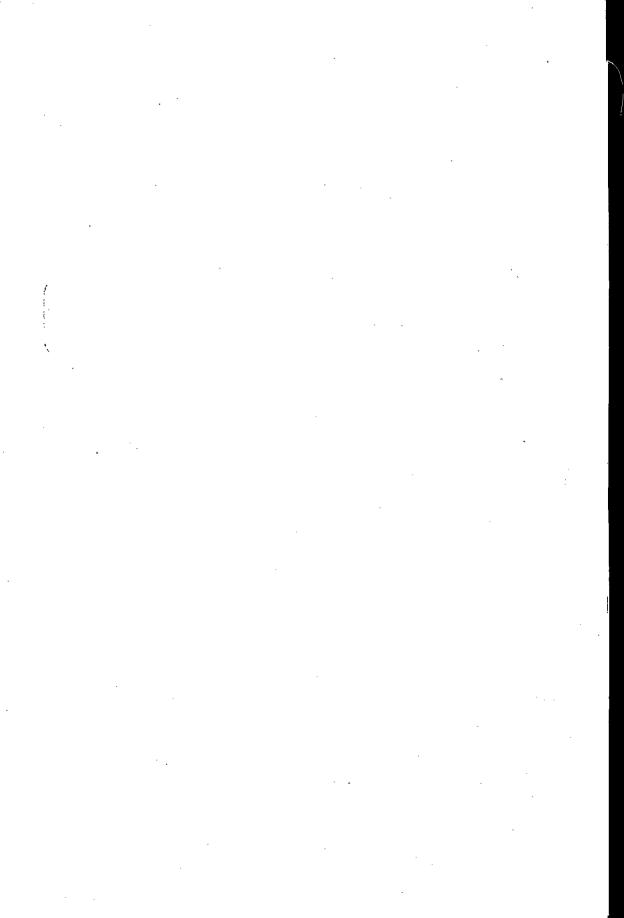
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ABSTRACT

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The main purpose of this thesis is to find a generalization for Harten's theorem for total variation non-increasing methods in the case of five, seven up to 2m+1 points for non zero integer m. Also, we present a general form of two-level and 2m-order in space and time explicit finite difference scheme with 2m+1-point for hyperbolic conservation laws. The form of this method is suitable for calculating the flux limiter technique for accuracy up to 2m-order. Also it will obtain the high resolution, total variation non increasing oscillations free of fourth, sixth and eighth order accurate explicit methods in space and time by adding suitable number of limiters of antidiffusive flux to a first order scheme. By the same way we shall treat the oscillations in the second and fourth order accurate implicit methods in the space and time. And the right oscillations are treated of these methods by adding inverse limiters for the limiters which we are adding to treat the left oscillations. The CFL condition is still satisfied. Also it presents the modification scheme for these methods to give high accuracy in the region of the discontinuities.

<u>Key words</u>: hyperbolic conservation laws, weak solutions, Riemann's problem entropy condition, monotone numerical methods, high resolution methods, flux limiter methods.



Contents

Contents

$oldsymbol{\mathrm{I}}$	age
SUMMARY	i
CHAPTER 1	
Hyperbolic Conservation Laws	
1.1 Introduction to Hyperbolic System of Partial Differential Equations.	1
1.2 The Need for Weak Solution of Hyperbolic Conservation Laws.	4
1.3 The Riemann Initial-Value Problem.	10
1.4 Entropy Condition for Weak Solutions.	13
1.5 Finite Difference Approach for Hyperbolic Conservation Laws:	15
1.5.1 Some Basic Concepts.	15
1.5.2 Numerical Entropy Stability.	18
1.5.3 Examples of Conservative Methods.	19
CHAPTER 2	
On Efficient Simpler Numerical Schemes for the Boundary Points f	or
Hyperbolic Conservation Laws of Riemann Problem.	
2.1 Introduction and the Proposed Efficient Simple Methods:	24
2.1.1 Upwind Scheme.	26
2.1.2 Lax-Friedrichs Scheme.	28
2.1.3 Lax-Wendroff Scheme.	30
2.1.4 Beam-Warming Scheme.	32
2.2 Numerical Examples and Comparison of Finite-Difference Methods	34
2.2.1 Analysis and Results for Different Courant's Number	34

CHAPTER 3

Nev	v Condition	nally Stable Exp	blicit Finite Difference Higher Order	
Sch	emes for	Hyperbolic Cons	ervation Laws	
3.1	Introduct	on.		39
3.2	New Exp	icit Finite-Differe	ence Schemes:	39
	3.2.1	Construction of Higher-Order Schemes		40
		3.2.1.1 Second	-order scheme.	44
		3.2.1.2 Fourth-	order scheme.	45
		3.2.1.3 Sixth-c	order scheme.	47
		3.2.1.4 Eighth-	order scheme.	48
	3.2.2	Stability Conditi	on For the Considered Schemes:	51
		3.2.2.1 Second	l-order scheme.	51
	."	3.2.2.2 Fourth-	order scheme.	51
		3.2.2.3 Sixth-c	order scheme.	52
		3.2.2.4 Eighth-	order scheme.	54
3.3	Example	and Comparative	e Cases:	55
	3.3.1	Smooth Initial Boundary-Value Problems.		
	3.3.2	Smooth Initial-Value Problems.		
	3.3.3	Riemann Problem.		
3.4	Numerica	l Results.		60
CH	APTER 4			,
Rec	ent Appr	oach of High Res	solution Methods for Hyperbolic	
Co	nservatio	Laws of Riema	nn Problem	-
4.1 Introduction.				74
4.2	High Res	olution Methods:		75
	4.2.1	Total Variation	Property and Convergence Theorems	75
	4.2.2	Flux Limiter Me	ethods.	. 80