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Some Problems on the Computational Mathematical Modelling

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Submitted to:

The Department of Mathematics, Faculty of Education,
Ain Shams University

By

Hussein Abd Allah Soliman Hussein

Supervised by

Prof. Dr. Ahmed Younis Ghaly

Professor of Applied Mathematics
Department of Mathematics
Faculty of Education
Ain Shams University

Prof. Dr. Hassan Nasr Ahmed Ismail

Retired Professor of Pure Mathematics
Department of Basic Science
Faculty of Engineering
Banha University

Dr. Emad Hassan Aly

Lecturer of Applied Mathematics
Department of Mathematics
Faculty of Education
Ain Shams University

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Mathematics Department

Candidate: Hussein Abd Allah Soliman Hussein

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Supervisors:

No.	Name	Profession	Signature
1.	Prof. Dr. Ahmed Younis Ghaly	Professor of Applied Mathematics, Mathematics Department, Faculty of Education, Ain Shams University.	
2.	Prof. Dr. Hassan Nasr Ahmed Ismail	Retired Professor of Pure Mathematics, Department of Basic Science, Faculty of Engineering, Banha University.	
3.	Dr. Emad Hassan Aly	Lecturer of Applied Mathematics, Mathematics Department, Faculty of Education, Ain Shams University.	



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بعض المشكلات في النمذجة الرياضية الحسابية

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مقدمة من الباحث

حسين عبدالله سليمان حسين

تحت إشراف

أ.د. حسن نصر احمد اسماعيل

أستاذ الرياضيات البحتة
قسم الرياضيات
كلية الهندسة ببنها- جامعة بنها

أ.د. أحمد يونس غالي

أستاذ الرياضيات التطبيقية
قسم الرياضيات
كلية التربية جامعة عين شمس

د. عماد حسن علي

مدرس الرياضيات التطبيقية
قسم الرياضيات
كلية التربية جامعة عين شمس

Abstract

This thesis consists of four chapters distributed as follows:

Chapter 1: It contains the most important basic concepts that will be used in this thesis

The goal of Chapter 2 is to study the effect of porous medium on the steady flow over a rotating disk with heat transfer. In this study the governing equations are transformed into nonlinear ordinary differential equations by applying the Von Karman [70] similarity transformation. Moreover, the resulting equations are then solved numerically by the finite difference method. Furthermore, numerical and graphical results for the velocity and temperature profiles are presented and discussed for various parametric conditions. Finally, comparisons with previously published works are performed and showed that the present results apply with White's results [71].

The purpose of Chapter 3 is to study steady flow over a rotating disk in porous medium with heat transfer and radiation effect. The Von Karman [70] similarity transformation is applied to transform the governing equations into nonlinear ordinary differential equations. In addition, the resulting equations are then solved numerically by the finite difference method. Moreover, numerical results are presented the distribution of velocity, and temperature profiles. The effects of varying the Prandtl number, the radiation parameter and porosity parameter are determined. Furthermore, comparisons the present results with

Childs's results [14], Rahman's results [51] and Owen and Rogers's results [49] are showed that the present results have high accuracy and are found to be an excellent agreement. At the end of this chapter the conclusions are summarized. The work in this chapter is preparing to submission.

Finally the main aim of Chapter 4 is to study the steady flow over a rotating disk in porous medium with heat transfer and magnetic effect. In addition, the governing equations are transformed into nonlinear ordinary differential equations by applying the Von Karman [70] similarity transformation and then solved numerically by applying the finite difference method. Moreover, the solutions are found to be governed by four parameters, the magnetic parameter M^* , the porosity parameter M , the Prandtl number Pr and the thermal radiation parameter R_d . We have compared the present radial, tangential and axial velocities and temperature distribution with the previously published work Anjali Devi et. al. [3], at different values of the M^* , M , R_d and Pr parameters. Furthermore, numerical and graphical results for the velocity and temperature profiles are presented and discussed for various parametric conditions. Finally, the conclusion is summarized.

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Chapter 1

Introduction

The introductory chapter is considered as a background for the material included in the thesis. The purpose of this chapter is to present a short introduction on the fluid mechanics, a brief survey of fluid properties and the basic flow equations. Moreover, it contains a short survey of some needed concepts of the material used in this thesis with a great of many enrichment details.

1.1 Fluid mechanics

Fluid mechanics deals with the study of all fluids under static and dynamic situations. Fluid mechanics is a branch of continuous mechanics which deals with a relationship between forces, motions, and statical conditions in continuous material. In fact, almost any action a person is doing involves some kind of a fluid mechanics problem. Furthermore, the boundary between the solid mechanics and fluid mechanics is some kind of gray shed and not a sharp distinction (see Figure (1.1) for the complex relationships between the different branches which only part of it should be drawn in the same time). Fluid mechanics is the study of fluids and the forces on them (fluids include liquids, gases, and plasmas). It can be divided into fluid kinematics, the study of fluid motion, and fluid dynamics, the study of the effect of forces on fluid motion, which can further be divided into fluid statics, the study of fluids at rest, and fluid kinetics, the study of fluids in motion. It is a branch of continuum mechanics, a subject which models matter

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without using the information that it is made out of atoms, that is, it models matter from a macroscopic viewpoint rather than from a microscopic viewpoint. The science of fluid mechanics has matured over the last 200 years, but even today we do not have complete and exact solutions to all possible engineering problems. Although the governing equations (called the Navier-Stokes equations) were established by the mid-1800s, solutions did not follow immediately. The main reason is that it is close to impossible to analytically solve these nonlinear partial differential equations for an arbitrary case. Consequently, the science of fluid mechanics has focused on simplifying this complex mathematical model and on providing partial solutions for more restricted conditions. Therefore the different chapters on classical fluid mechanics are based on retaining different portions of the general equation while neglecting other lower-order terms. This approach allows the solution of the simplified equation, yet preserves the dominant physical effects (relevant to that particular flow regime). Finally, with the enormous development of computational power in the 21st century, numerical solutions of the fluid mechanic equations have become a reality. However, in spite of these advances, elements of modeling are still used in these solutions, and the understanding of the “classical” but limited models is essential for successfully using these modern tools [32].

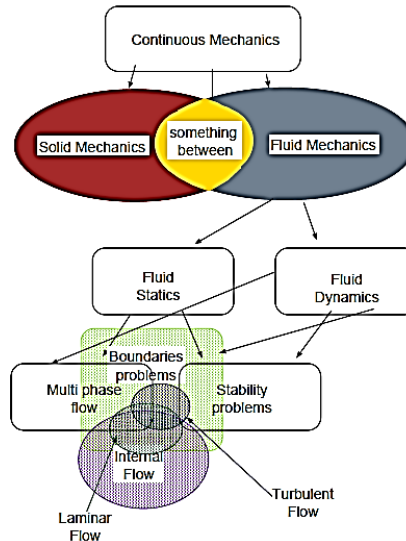


Figure 1.1: Diagram to explain part of relationships of fluid mechanics branches.

1.2 Types of fluid flow

There are many types of fluid, some of them are mentioned in the following subsection.

1.2.1 Compressible and incompressible flow

The terms compressibility and incompressibility describe the ability of molecules in a fluid to be compacted or compressed (made more dense) and their ability to bounce back to their original density, in other words, their “springiness”. An incompressible fluid cannot be compressed and has relatively constant density throughout. A compressible flow is a situation in which the density of the fluid cannot be assumed to be constant [10]. It is the area of fluid mechanics that deals with fluids in which the fluid density varies significantly in response to a change in pressure. Compressibility effects are typically considered significant if the mach number (the ratio of the flow velocity to the local speed of sound) of the flow exceeds 0.3, or if the fluid undergoes very large pressure changes. In fluid mechanics or more generally continuum mechanics, incompressible (isochoric) flow refers to flow in which the material density is constant within an infinitesimal volume that moves with the velocity of the fluid. An equivalent statement that implies incompressible flow is that the divergence of the fluid velocity is zero [10]. Incompressible flow does not imply that the fluid itself is incompressible. Incompressible fluids must have a constant density everywhere, while incompressible flow only requires that the density remain constant within a parcel of fluid which moves with the fluid velocity [11].

1.2.2 Uniform and non-uniform flow

The classification of the fluid flow based on the variation of the flow parameters with distance or space. It characterizes the flow as uniform or non-uniform. The fluid flow is a uniform flow if the flow parameters remain constant with distance along the flow path. And the fluid flow is non-uniform if the flow parameters vary and are different at different points on the flow path. For a uniform flow, by its definition, the area

of the cross section of the flow should remain constant. So a fitting example of the uniform flow is the flow of a liquid thorough a pipeline of constant diameter. And contrary to this the flow through a pipeline of variable diameter would be necessarily non-uniform [72].

1.2.3 Steady and unsteady flow

The other classification criterion for the fluid flow is based on the variation of the fluid flow parameters with time characterizes the flow in two categories, steady and unsteady flow. If the flow parameters, such as velocity, pressure, density and discharge do not vary with time or are independent of time then the flow is steady. If the flow parameters vary with time then the flow is categorized as unsteady [63]. Steady flows are often more tractable than otherwise similar unsteady flows. The governing equations of a steady problem have one dimension fewer (time) than the governing equations of the same problem without taking advantage of the steadiness of the flow field.

A steady flow can be uniform or non-uniform and similarly an unsteady flow can also be uniform or non-uniform.

1. Steady and Uniform Flow: flow through a pipeline of constant diameter with a discharge constant with time.
2. Steady and non-uniform flow: fixed discharge flow through a tapering pipe. Water flow through a river with a constant discharge is also a good example of such flow as the span of river generally varies with distance and amount of water flow in river is constant.
3. Unsteady and uniform flow: a flow through pipeline of constant cross section with sudden changes in fluid discharge or pressure.
4. Unsteady and non-uniform flow: pressure surges in a flow through a pipe of variable cross section. A practical example can be the water flow in the network of canals during water release.

1.2.4 Laminar flow

Laminar flow, sometimes known as streamline flow, occurs when a fluid flows in parallel layers, with no disruption between the layers

[11]. At low velocities the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards. There are no cross currents perpendicular to the direction of flow, nor eddies or swirls of fluids. In laminar flow the motion of the particles of fluid is very orderly with all particles moving in straight lines parallel to the pipe walls. In fluid dynamics, laminar flow is a flow regime characterized by high momentum diffusion and low momentum convection. Laminar flow is the opposite of turbulent flow which occurs at higher velocities. The dimensionless Reynolds number is an important parameter in the equations that describe whether flow conditions lead to laminar or turbulent flow. In the case of flow through a straight pipe with a circular cross-section, Reynolds numbers of less than 2100 are generally considered to be of a laminar type [48].

1.2.5 Turbulent flow

In fluid dynamics, turbulence or turbulent flow is a flows with high Reynolds numbers usually become turbulent, while those with low Reynolds numbers usually remain laminar. For pipe flow, a Reynolds number above about 4000 will most likely correspond to turbulent flow, while a Reynolds number below 2100 indicates laminar flow. The region in between ($2100 < Re < 4000$) is called the transition region. In turbulent flow, unsteady vortices appear on many scales and interact with each other [32]. There are many examples of turbulence such as:

- Smoke rising from a cigarette is turbulent flow. For the first few centimeters, the flow is certainly laminar. Then smoke becomes turbulent as its Reynolds number increases.
- Most of the terrestrial atmospheric circulation.
- The external flow over all kind of vehicles such as cars, airplanes, ships and submarines.

1.3 Fluid properties

Fluids, in general, may have many properties related to thermodynamics, mechanics, or other fields of science. In the following subsec-

tions, only a few, which are used in introductory fluid mechanics, are mentioned.

1.3.1 Density

The density is a property which requires that liquid to be continuous. Different materials usually have different densities, so density is an important concept regarding buoyancy, purity and packaging. The density can be changed and it is a function of time and space (location) but must have a continuous property. It doesn't mean that a sharp and abrupt change in the density cannot occur. It referred to density that is independent of the sampling size. Figure 1.2 shows the density as a function of the sample size. After certain sample size, the density remains constant. Thus, the density is defined as $\rho = \lim_{\Delta V \rightarrow \epsilon} \frac{\Delta M}{\Delta V}$.

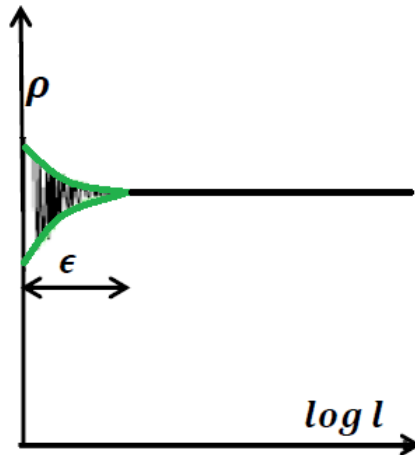


Figure 1.2: Density as a function of the size of sample.

It must be noted that ϵ is chosen so that the continuous assumption is not broken, that is, it did not reach/reduced to the size where the atoms or molecular statistical calculations are significant (see Figure (1.2) for point where the grey lines converge to constant density). When this assumption is broken, then, the principles of statistical mechanics must be utilized [11].

1.3.2 Newtonian fluid

A Newtonian fluid is a fluid whose stress versus strain rate curve is linear and passes through the origin [71]. The constant of proportionality is known as the viscosity. For example, water is Newtonian, because it continues to exemplify fluid properties no matter how fast it is stirred or mixed. For a Newtonian fluid, the viscosity, by definition, depends only on temperature and pressure of the fluid not on the forces acting upon it [63]. A simple equation to describe Newtonian fluid behavior is

$$\tau = \mu \frac{du}{dy} \quad (1.1)$$

where τ is the shear stress exerted by the fluid, μ is the fluid viscosity and constant of proportionality $\frac{du}{dy}$ is the velocity gradient perpendicular to the direction of shear [34].

1.3.3 Non-Newtonian fluid

A non-newtonian fluid is a fluid whose flow properties differ in any way from those of newtonian fluids. Most commonly the viscosity of non-newtonian fluids is not independent of shear rate or shear rate history. However, there are some non-newtonian fluids with shear-independent viscosity, that nonetheless exhibit normal stress-differences or other non-newtonian behavior [71]. Many salt solutions and molten polymers are non-newtonian fluids, as are many commonly found substances such as ketchup, custard, toothpaste, starch suspensions, paint, blood, and shampoo [10]. In a newtonian fluid, the relation between the shear stress and the shear rate is linear, passing through the origin, the constant of proportionality being the coefficient of viscosity. In a non-newtonian fluid, the relation between the shear stress and the shear rate is different, and can even be time-dependent. Therefore a constant coefficient of viscosity cannot be defined [32].

1.3.4 Viscosity

Viscosity is a measure of the resistance of a fluid which is being deformed by either shear stress or tensile stress. viscosity is “thickness” or “internal friction”. Thus, water is “thin”, having a lower viscosity,

1.3. FLUID PROPERTIES

while honey is “thick”, having a higher viscosity. The less viscous the fluid is, the greater its ease of movement (fluidity) [66]. There exist a number of forms of viscosity as shown in Figure (1.3).

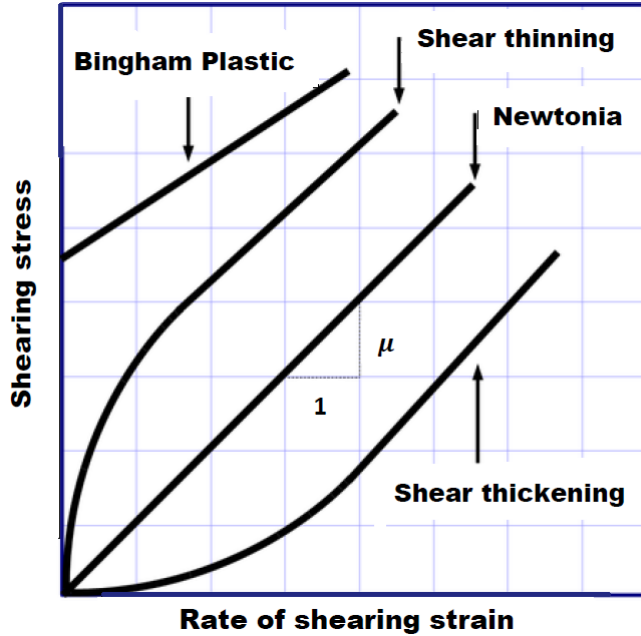


Figure 1.3: Types of viscosity

- Newtonian: fluids, such as water and most gases which have a constant viscosity.
- Shear thickening: viscosity increases with the rate of shear.
- Shear thinning: viscosity decreases with the rate of shear. Shear thinning liquids are very commonly, but misleadingly, described as thixotropic.
- Thixotropic: materials which become less viscous over time when shaken, agitated, or otherwise stressed.
- Rheopectic: materials which become more viscous over time when shaken, agitated, or otherwise stressed.