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Reliability Sampling Plans for the Pareto Lifetime Model under Progressive Censoring

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To the memory of my grandmother and my aunt

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Abstract

Quality has become one of the most important consumer decision factors in the selection from among competing products and services. Acceptance sampling is one of the major components of the field of statistical quality control. Sampling plans which determine the acceptability of a product with respect to its lifetime are called reliability sampling plans. In this study, the design of reliability sampling plans for the Pareto lifetime model under progressive Type II censoring is considered. The analysis is carried out within the Bayesian framework for progressively Type II censored samples with random removals, where the number of units removed at each failure follows a binomial distribution. Sampling plans are derived using the decision theory approach with a suitable cost function that consists of the sampling cost, the rejection cost, and the acceptance cost. The decision rules are based on the estimated reliability function and the total time on test. Simulations for evaluating the Bayes risk are carried out and the optimal sampling plans are reported for various sample sizes, observed number of failures and binomial removal probabilities. In addition, a life test acceptance criterion is developed using the prediction interval approach suggested by Fertig and Mann (1977).

Key Words

Progressive censoring — *Reliability sampling plans*

Pareto distribution — *Bayesian prediction intervals*

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Summary of the thesis

Acceptance sampling is one of the major components of the field of statistical quality control. It is the procedure adopted by taking samples from batches or lots of some industrial products, inspecting these samples, and on the basis of the inspection results deciding whether the whole batch or lot is accepted or rejected. Designing a sampling plan involves determining the sample size and the acceptance criteria based on which the lot is accepted or rejected. Such samples may be complete or censored. In reliability and life testing studies, inference is usually based on a censored rather than a complete sample.

Censoring is commonly used in life tests because of time limits and other restrictions on data collection. Censoring arises in various ways, for example, in Type I censoring, the units are initially placed on test and the experimentation terminates after a prespecified time. On the other hand, in Type II censoring, the test is terminated when a predetermined number of failures is observed. In many life testing experiments, units may be lost or removed from the experiment at points other than the final termination point. Such loss may either occur unintentionally, as in the case of accidental breakage of an item, or it may be pre-planned so as to save time and cost or to use the removed live units in other experiments. In such cases, progressive, rather than conventional, Type I or Type II censoring is usually used as it allows for the removal of experimental units before the end of the test.

In this study, the design of reliability sampling plans for the Pareto lifetime model under progressive Type II right censoring is considered. The Pareto distribution was first introduced by Pareto in 1897 for the distribution of income. Then, it has found widespread use as a model for various socio-economic phenomena. Furthermore, the Pareto distribution has been used in reliability and lifetime modeling.

This study is divided into four chapters and two appendices. Chapter 1 provides an overview of some sampling plans schemes and introduces the concept of progressive censoring. In addition, it gives an overview of the Pareto lifetime model and provides a review of the literature on the proposed topic.

In Chapter 2 an overview of some of the basic Bayesian concepts and an illustration of the suggested progressive Type II censoring scheme are given. Moreover, the loss function and the decision rules used to derive the reliability sampling plans under the decision theory approach are presented. Furthermore, the three density estimation techniques; the histogram, the kernel, and the adaptive kernel approaches, used to design the Bayesian sampling plans are introduced.

Chapter 3 presents a simulation study that is used to evaluate the Bayes risk and hence, achieve the optimal sampling plans. This simulation process involves data generation, density estimation, evaluation of the Bayes risk, and derivation of the optimal sampling plans. Moreover, a discretization method that is used to obtain the discretized prior distributions of the shape and scale parameters is illustrated.

Chapter 4 presents a life test acceptance criterion developed under the prediction interval approach suggested by Fertig and Mann (1977). The analysis is also carried out within the Bayesian context. The predictive distributions and the corresponding prediction intervals for the $(h+1)^{th}$ ordered lifetime of the unsampled portion of the lot are derived. Finally, the main results of the study as well as some points that may be worthy of future research are pointed out.

The MATLAB and *R* codes that are used in the simulation study discussed in Chapter 3 are provided in Appendix 1. In addition, the tables reporting the values of the Bayes risk and the corresponding sampling plans derived from the simulation study for certain binomial removal probabilities are given in Appendix 2

Contents

List of Tables

Table (A.18) Bayes risk and the corresponding sampling plans for $p = 0.7$ and $P(Q \geq q_0)$ estimated using the adaptive kernel approach

List of Figures

Introduction

Acceptance sampling is one of the major components of the field of statistical quality control. It is the procedure adopted by taking samples from batches or lots of some industrial products, inspecting these samples, and on the basis of the inspection results deciding whether the whole batch or lot is accepted or rejected. An important quality characteristic is the lifetime of the product; that is, the duration during which the item successfully performs its intended function under the specified conditions. Sampling plans which determine the acceptability of a product with respect to its lifetime are called reliability sampling plans; Schneider (1989).

Designing a sampling plan involves determining the sample size and the acceptance criteria based on which the lot is accepted or rejected. Such samples may be complete or censored. In reliability and life testing studies, inference is usually based on a censored rather than a complete sample. Censoring is commonly used in life tests because of time limits and other restrictions on data collection. A sample is said to be censored when the exact lifetimes are known for only a portion of the units under study. Censoring arises in various ways, for example, in Type I censoring, the units are initially placed on test and the experimentation terminates after a prespecified time. On the other hand, in Type II censoring, the test is terminated when a predetermined number of failures is observed.

In many life testing experiments, units may be lost or removed from the experiment at points other than the final termination point. Such loss may either occur unintentionally, as in the case of accidental breakage of an item, or it may be pre-planned so as to save time and cost or to use the removed live units in other experiments. In such cases, progressive, rather than conventional, Type I or Type II censoring is usually used as it allows for the removal of experimental units before the end of the test.

In reliability studies, data are usually modeled using distributions with positive support such as the exponential, Weibull or Pareto distributions. The Pareto distribution was first introduced by Pareto (1897) for the distribution of income, as mentioned in Arnold and Press (1983). Then, it has found widespread use as a model for various socio-economic phenomena. Furthermore, the Pareto distribution has been used in reliability and lifetime modeling, see, for example, Berger and Mendelbort (1963), Harris (1968) and Davis and Feldstein (1979).

In this study, the design of reliability sampling plans for the Pareto lifetime model under progressive Type II right censoring is considered. Sampling plans are derived using the decision theory approach with a suitable loss or cost function that consists of the sampling, rejection, and acceptance costs. The decision rules are based on the estimated reliability function and the total time on test. Plans are constructed within the Bayesian context using natural conjugate priors. Simulations for evaluating the Bayes risk are carried out and the optimal sampling plans are reported for various sample sizes, observed number of failures and removal probabilities. In addition, a life test acceptance criterion is developed using the prediction interval approach suggested by Fertig and Mann (1977). The analysis is carried out within the Bayesian framework for progressively Type II censored samples with random removals, where the number of units removed at each failure follows a binomial distribution.

Chapter 1 provides an overview of some sampling plans schemes, introduces the concept of progressive censoring, gives an overview of the Pareto lifetime model and provides a review of the literature on the proposed topic. In Chapter 2 an overview of some of the basic Bayesian concepts and an illustration of the suggested censoring scheme are given. The loss function, the decision rules used to derive the reliability sampling plans under the decision theory approach, and the three density estimation methods that are used to design the sampling plans are also introduced. Chapter 3 presents a simulation study that is used to evaluate the Bayes risk and hence, achieve the optimal sampling plans. Moreover, a discretization method that is used to obtain the discretized prior distributions of the shape and scale parameters is illustrated. In Chapter 4, a life test acceptance criterion is developed under the prediction interval approach suggested by Fertig and Mann (1977). The predictive distributions and the corresponding prediction intervals for the $(h+1)^{th}$ ordered lifetime of the unsampled portion of the lot are derived. An illustrative example of the proposed criterion is given. Finally, the main results of the study as well as some points that may be useful for future research are pointed out.

The results presented in this study are derived using programs written in MATLAB and *R* software packages. These codes are given in Appendix 1. It is worth mentioning that these codes can be easily modified to fit other distributions, parameter settings or input values.

Chapter 1

Basic concepts and literature review

1. Introduction

Quality has become one of the most important consumer decision factors in the selection from among competing products and services, whether the consumer is an individual, an industrial organization, or a retail store. Moreover, there is a substantial return on investment from improved quality and from successfully employing quality as an integral part of overall business strategy.

In the 1930s and 1940s, acceptance sampling was one of the major components of the field of statistical quality control, and was primarily used for incoming or receiving inspection; Montgomery (2001). Acceptance sampling is the procedure adopted by taking samples from batches or lots of some industrial products, testing or inspecting these samples, and on the basis of the inspection results deciding whether the whole batch or lot is satisfactory, and hence accepting or rejecting this lot. These products may be raw materials, manufactured components, or finished products. Designing a sampling plan involves determining the sample size and the acceptance criteria based on which the lot is accepted or rejected. Acceptance sampling procedures are most useful in case of destructive tests, when the cost of inspection is quite high, or when inspection requires much time or involves many personnel.

In this study, we are mainly concerned with applying acceptance sampling techniques on the Pareto lifetime model under progressively Type II censored data. An overview of some acceptance sampling techniques is given in Section 2. Section 3 introduces the concept of progressive censoring, and specifically, progressive Type II right censoring. An overview of the Pareto lifetime model is given in Section 4. A review of the literature on the proposed topic is provided in Section 5. The main objectives of the study are given in Section 6.