



# **PROBABILISTIC ANALYSIS OF SOME DIFFERENT REDUNDANT SYSTEMS**

## **THESIS**

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## **PREFACE**

The design incorrect manufacturing techniques are obvious reasons of the low reliability. Some manufactures hesitate to invest more money on an improved design and modern techniques of manufacturing and testing. Improper selection of materials is another cause for poor design. Components equipments do not operate in the same manner in all conditions. A complete knowledge of their characteristics, applications, limitations will avoid their misuse and minimize the occurrence of failures.

Accordingly, the main purpose of this thesis is to maximize the reliability of the system, so we study some different redundant systems. The system considered may be consisted of similar redundant units. The systems may have different types of operation (repair only, repair, replaced and preventive maintenance), the systems may have different types of failures (human errors, partial hardware failure, common-cause failure and total hardware failure). Using both the modern methods and the computer program to compute the several measures of reliability are obtained such as mean time to system failure, steady-state availability, and variance of time to failure, probability that the repairman is busy.



This thesis consists of six chapters.

**This first chapter** presents some definitions, basic characteristics and some concepts for the reliability theory. Also the reliability function, the hazard rate, the cumulative hazard function and the mean residual life function, combinatorial aspects are defined. We note definition of failure and different types of failures (human errors, partial hardware failure, common-cause failure and total hardware failure). The Laplace transforms, system of linear first orders differential equations, standby redundant system and the Markov models are defined. Also we taking some examples and graphs to showing the basic characteristics.

**The Second chapter** discusses a system with two unit worm standby system having one expert repairman and one assistant repairman. The assistant is called only if the expert repairman is busy. The expert repairman gives instructions to his assistant if the assistant repairman needs for completing the repair. Techniques of the semi-Markov processes and regenerative processes are used to obtain various measures of system effectiveness and profit incurred.

Using a regenerative point technique for the Markov-renewal process, to calculate the mean time to failure, the steady state availability , the profit analysis, the busy period of an expert repairman and assistant repairman , have been shown by performing comparisons numerically and graphically by using the computer programs. The study performed in this chapter has been published in “African Journal of Computer Science Research Vo1.6(2), pp.20-25,(2013)”.

**The third chapter** considers a repairable redundant the failed units can either be repaired or replaced by identical standby to reduce the system down time. The failed units are inspected for repair/replacement. In this chapter, one stochastic model for 2-out-of-3 redundant system of identical units with repair and inspection are examined stochastically. the system is considered in up-state only if 2-out-of-3 units are operative in this model. Normally, the server either attends the system promptly or may take some time, after failure. The system is studied under an operational restriction on the inspection i.e. in case when system has only one unit in operational mode the server has to attend the system for inspection. Semi- Markov processes and regenerative point

technique is adopted to obtain the expressions for measures of system effectiveness such as transition probabilities, mean sojourn times, to system failure, steady state availability, busy period, expected number of visits etc. Cost-analysis is also carried out for the system model. Certain important results have been derived as particular cases. The study performed in this chapter send to publish in Journal of Advances and Applications in Statistical Sciences.

**The fourth chapter** deals with the probabilistic analysis of a system consisting of a single unit operated by a person who may be in good or poor physical condition which also affect the performance of the system. The failure and repair times of the unit and change times physical conditions of the operator are assumed to have different arbitrary distributions. Explicit expressions for the mean time to system failure pointwise availability, steady state availability, busy period by the server, expected number of visits by the server and cost per unit time have been obtained. One special case was studied in which the failure and repair are assumed to be exponentially distributed. The results given previously have been derived as special case from the results of this chapter. Computer

programs to calculate the mean time to system failure for the system has been done.

**The fifth chapter** presents a stochastic analysis of a dissimilar two-unit parallel system with preventive maintenance and common-cause failure. The system goes for preventive maintenance at random epochs.

A bivariate extension of the exponential distribution is taken for the lifetimes of two components. The failure, replacement and maintenance time's rates are assumed to the constants for the units. Using the theory of regenerative and Markov-renewal processes several measures of reliability are obtained. Certain important results have been derived as particular cases. By using computer programs some numerical results of steady-state availability, mean-time to system failure and profit analysis are calculated. The plots for system reliability, system mean time to failure and system availability are drawn for various parametric values. The effect of preventive maintenance on the system performance is shown graphically. The results obtained by Coel and Preeti (1992) are derived from the present chapter as particular cases.

**The sixth chapter** deals with a stochastic analysis of two unit parallel system focuses on repair and/or replacement of the entire unit/system. However this system is exposed to various degree of deterioration which can result in dangerous and costly failure. Where such deteriorations occurred, minor maintenance and major maintenance are done to the system. Less attention was given on the effect of deterioration, minor and major maintenance on various measures of system effectiveness such as mean time to system failure and system availability. The objective of this paper is to present the effect of deterioration and minor maintenance rates on mean time to system failure. We analyzed the system by using Kolmogorov's forward equations method. Effect of deterioration and minor maintenance rates on MTSF(mean time system failure) have also been discussed graphically. Results have shown that a deterioration rate reduces the MTSF while minor and major maintenance improves the MTSF. Through combine action of minor maintenance and major repair action the MTSF is improved. Models presented in this chapter are important to engineers and maintenance managers, for proper maintenance analysis, decision and for safety of the system as a whole.

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# **CHAPTER ONE**

## **BASICS AND CONCEPTS FOR RELIABILITY THEORY**

### **1.1 INTRODUCTION**

Since time immemorial humanity treasured reliability as a very important human attributes. However, a judgment can be made whether an individual is reliable or not on the basis of a definite human function. For instance the reliability of an individual working with an organization may be assessed on the basis of punctually of arriving at work or in attending meetings.

Generally reliability in a wider sense may be considered as a measure of performance. Persons who complete their tasks on schedule are described as reliable because they are able to finish their work on time. Reliability of human beings therefore depends on time at which or in which they perform any particular task which may be taken as a measure. Reliability dose not only apply to man's actions but also to the objects he uses. We have seen that reliability has been applied to man's