Evaluation of Sommerfeld Integrals for Half-Space Problems

A THESIS

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SUMMARY

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This Monograph gives a systematic and unified treatment of the electromagnetic field in the upper and lower half - spaces separated by a plane interface due to a dipole radiating in the upper half - space (air).

This is presented in three chapters as follows:

In chapter one it is illustrated, in a concise way, the coordinate and fundamental equations of the field in an infinite, homogeneous, and isotropic medium. Moreover, it includes a review of some topics about Hertz and Fitzgerald vectors, the concepts of the nature of the dipole source, and the construction of suitable integral representations for the cylindrical components of the Hertz and Fitzgerald vectors/ Hankel transformation, as a guide, was applied in order to obtain integral representations for the Hertz and Fitzgerald vectors that satisfy the Helmholtz wave equation for a time factor exp(+iωt), as is well known. A particular solution of the nonhomogeneous wave equation is obtained firstly by the infinite space Green's function, and it is expressed as an integral in terms of the so-called Sommerfeld integral representation. Secondly, a solution of the homogeneous wave equation as a linear combination of the products of exponential and Bessel functions is deduced. Therefore the Hertz and Fitzgerald

vectors in each space can be obtained using Sommerfeld integrals. These results help in obtaining the integral representations of the electromagnetic field in each space.

The non-uniqueness of the field representation in terms of Debye potentials is stressed and a proper representation for horizontal dipoles is used.

In chapter two the radiation and far - zone fields in Wait's concepts are reviewed. The evaluation approximation of the Sommerfeld integrals was applied for the Hertz vector of the vertical electric dipole in the upper half - space. The agreement is excellent when the ratio of wave numbers characteristic of the denser half - space and air is large. A general formula is deduced which is valid everywhere, from which Wait's formula (1985) can be obtained as a special case for points of observation close to the interface. Also, the physical meanings for some numerical calculations which show the effect at the far distances from the boundary on the field are discussed.

In chapter three simple formulas are derived as an approximation for the vertical components of the electric field generated by a horizontal electric dipole in air. The evaluation is carried out and is subject to the condition requirement that the wave number of the lower half - space would be much greater in magnitude than the upper half - space (in which the antenna is located).

In order to check the accuracy of the newly derived simple formulas of this work, we have compared the present results with those of King and Wu(1982), also they are represented graphically.

INTRODUCTION

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The purpose of the present work is to discuss in detail systematic and unified treatment of the electromagnetic field in the two half - space problem, which is bounded by a horizontal plane interface separating the upper-half dielectric space (air) from the homogeneous and isotropic lower half - space (ground, salt water, fresh water, etc.,), where the field is due to source dipoles in air.

The main features of the study depend upon making use of a suitable Hankel transformation. Thus, the two - media boundary value problem is reduced to the evaluation of two fundamental integrals. These integrals help in computing the electromagnetic field components for all pertinent cases of the dipole radiation, where both the dipole and the point of observation lie in the air.

The underlying basis of the study is the numerical treatment of Sommerfeld integrals, where the evaluation accuracy of the integrals will be carried out subject to the condition that the wave number of the lower half - space would be much greater in magnitude than that of the upper half - space (in which the antenna is located), i.e., the ground conductivity is sufficiently large. Thus we present, for points of observation everywhere asymptotic expansions which are valid quite generally.

Various asymptotic solutions have been developed for different ranges of observation. Recent development concerning far-field evaluation was carried out by Wait (1985), who used the saddle - point technique which assumes large refractive index. As an extension of the problem we have already used another approximation to increase the range of the applicability of this technique. This is one of the main goals of the present study.

Historically, the boundary between two electrically different media such as air and earth greatly alters the nature of the electromagnetic waves travelling outward from an electric dipole near the surface. The original work on this subject started analytically by Sommerfeld [1,2] and appeared in a series of papers and articles that have served as the foundation for related investigations by others up to the present time. Thus, the integrals expressing the interaction of electromagnetic sources with the ground are known as Sommerfeld integrals. The solution obtained by Sommerfeld was for the Hertz potential of the two half - space model (earth and air) with an oscillating vertical electric dipole on the boundary as the source. The effects of the earth's curvature and reflection from the ionosphere, which further complicate the actual propagation over the surface of earth, are not included. The components of the electric and magnetic fields are determined by differentiation.

Horschelman [3], a student of Sommerfeld, was the first researcher who has treated the horizontal electric dipole in air, in his doctoral dissertation. Employing Sommerfeld formulation of the problem, the vertical magnetic dipole in air was intially treated by Elias [4], who has introduced the Fitzgerald potential to describe the electromagnetic field. Sommerfeld [5]employing his formulationin the case ofcylindrical waves, was the first to treat all four cases of dipole sources (vertical and horizontal electric and magnetic dipoles) in air. Shortly afterwards, Strutt [6] also treated all four types of dipole sources, however, employing Weyl's formulation of the problem.

The radiation of a Hertzian dipole immersed in the conducting medium (lower half - space) appears to have been first studied by Tai [7], by an ingenius application of Sommerfeld's formulas.

Since then there have appeared numerous papers and reports dealing with dipole radiation arising from source dipoles embedded in the conducting medium. Among the early ones, is the case of magnetic dipole antenna which is immersed in a conducting medium that was treated by Wait [8]. Similar expressions are given by Wait and Campbell [9] in the case of a horizontal electric dipole similarly embedded in earth. They presented the electromagnetic field components for points of observation close to the interface separating the two media and for distinct ranges which are characterized as the near, intermediate and far field zones.