TREATMENT OF TRANSITIONS IN QUANTUM ELECTRODYNAMICS

THESIS

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بسم الله الرحمن الرحيم

TO MY PARENTS AND MY BROTHER

Whose love and encouragement were among the incentives in accomplishing this hard work.

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INTRODUCTION

In this thesis we use the second quantisation procedures in the four dimensional space to obtain the transition probability from a state of a given number of electrons, positrons and photons of given momenta and spins to other different given number of electrons, positrons and photons of given momenta and spin. It is known that in the usual treatment of non-relativistic quantum mechanics the electrons (or positrons) are treated as a one particle state, with possible interaction with an electromagnetic field. The photons in the electromagnetic field can not be treated on the same footing as the electron, as photons can be emitted or absorbed, when the electron passes from one energy state to another.(1),(2)

Then, a photon although it does not decay spontaneously it is continuously created or annihilated during the processes. Thus in the non-relativistic quantum mechanics the interaction of the electron (the stable particle) having several quantum states is with the electromagnetic field (and not with the photon). The electromagnetic field represented by the vector potential $A_\mu(x)$ is then considered as an operator including photon states of all possible momenta and spin each multiplied by what is known as creation and annihilation operators. Different from the electron quantum state, the electromagnetic field state is defined in terms of a number of photons of given momenta and spin. That is we have a many body state. One photon function of given momenta and spin is now defined as the matrix element of the operator A_μ (x) between the vacuum and an electromagnetic state of one photon created, expressed by a creation operator applied to the vacuum.

Now the relativistic electron-positron field is treated in the same way as the electromagnetic field.(3),(4) Thus we have also a many body treatment for the electron-positron field. The $\psi(x)$ statisfying by Dirac equation is now considered as an operator expressed as a sum of annihilation operators of electrons and creation operators of positrons, and its adjoint $\bar{\psi}(x)$ includes creation operators for electrons and annihilation operators for positrons. In the beginning of this thesis we shall explain briefly how these procedures are developed to obtain a given recipe for the transition probability. In the first chapter the lagrangian formalism, conservation laws from the equations of motion are given. The free electromagnetic field is then discussed. The vector potential A_μ (x) commutation relations (second quantisation rule) considering them as operators composed of annihilation and creation terms (5),(6) We also discuss the Dirac equations of motion, their commutation relations for the electron-positron field by using the operators $\psi(x)$, $\bar{\Psi}(x)$ containing annihilation and creation operators. The equations of motion for the Maxwell's and Dirac fields in interaction are next discussed.

The S-matrix formalism is given from the Schrödinger equation hence the transition probability expressed as the expectation value of products of field operators.

In the second chapter we give explicit expression of the n th order S-matrix element between the initial and final states of any numbers of electrons, positrons and photons of given momenta and spin. This is expressed as an integral in the x-space of an expression, given as a product of an electron-positron part multiplied by a photon part.

The electron part is given as a nxn determinant containing electron-position singular functions and their ingoing and outgoing functions. Where as the photon part consists of a single term containing their singular functions and their ingoing and outgoing functions. In this way one have a recipe to compute the n th order

S-matrix in any problem without thinking of the possible Feynman graphs for this problm, obeying the different symmetries.

In the third chapter the differential scattering cross section for the Bremstrahlung processe is discussed in details first as a function of the energies of the scattered electron and photon and their direction in space, that a function of (E,E',θ',Ø,θ) (Noticing that the energy of the photon w=E-E'). (7) Next integrating over all directions of the scattered electron (the angle θ',Ø) we obtain the differential scattering cross section for the emitted photon only (depending on E,E',θ), in the most simplified form. Detailed numerical results are given, as the discussion of such complicated expressions of the differential cross-section is not found in the litterature.

Finally integrating over all directions of the photon we obtain the total scattering cross section for the Bremstrahlung (depending only on E,E'). The chapter ends on obtaining an expression of the energy loss, and the numerical results for the different cross sections.

CAHPTER I

THE ELECTROMAGNETIC AND ELECTRON POSITRON FIELDS

CHAPTER I

THE ELECTROMAGNETIC AND ELECTRON POSITRON FIELDS

1.1 LAGRANGIAN FORMALISM:

Before the discussion of the different free fields, we shall first consider the general procedure to obtain the field equations of motion.

We shall adopt the Lagrangian Formalism

$$\int \mathcal{L}\left(\psi_\nu , \frac{\partial \psi_\nu}{\partial x_\mu} , \cdots \right) d^4x = \text{extremum} \qquad (1.1)$$

Where $\mathcal{L}$ is the Lagrangian given as a function of the fields ψ_ν (ν =1,2,......) and their derivatives $\frac{\partial \psi_\nu}{\partial x_\mu}$ (μ = 1,2,3,4). $\left(\int_{t1}^{t2} dt \int d^3x \, \mathcal{L} \right)$

The integral is taken over the four dimensional space. From the above condition, the equations of motion are then known to be

$$\frac{\partial \mathcal{L}}{\partial \psi_\nu} - \frac{\partial}{\partial x_\mu} \left(\frac{\partial \mathcal{L}}{\partial \left(\frac{\partial \psi_\nu}{\partial x_\mu}\right)} \right) = 0 \qquad (\nu = 1,2,3,4) \qquad (1.2)$$

Using these equations of motion, equation (1.1) can be expressed

$$\delta \int \mathcal{L} \, d^4x = \int d^4x \, \frac{\partial}{\partial x_\mu} \left(\delta \psi_\nu \frac{\partial \mathcal{L}}{\partial \left(\frac{\partial \psi_\nu}{\partial x_\mu}\right)} \right) \qquad (1.3)$$

The total momentum and the total angular momentum of the field is defined through the four dimensional energy momentum tensor $\Theta_{\mu\nu}$ of the field, and $G_{\mu\nu,\lambda}$ which will be defined later

$$P_{\mu} = -i \int \theta_{4\mu} \, d^3x \tag{1.4}$$

$$M_{\mu\nu} = -i \int G_{\mu\nu,4} \, d^3x \tag{1.5}$$

The four dimensional tensors $\theta_{\mu\nu}, G_{\mu\nu,\lambda}$ are obtained from the chosen Lagrangian using the invariance under displacement and rotation in the four dimensional space.

CONSERVATION LAWS:-

(i) Conservation of momentum energy four vector is obtained from the invariance under a displacement in the four dimensional space. Using the infinitismal displacement

$$x_{\mu} \longrightarrow x_{\mu} + \delta a_{\mu}$$

We deduce from the condition (1.3)

$$\frac{\partial}{\partial x_{\rho}} \theta_{\rho\mu} = 0 \tag{1.6}$$

where

$$\theta_{\rho\mu} = \delta_{\rho\mu} \mathcal{L} - \frac{\partial \psi_{\nu}}{\partial x_{\mu}} \frac{\partial \mathcal{L}}{\partial\left(\frac{\partial \psi_{\nu}}{\partial x_{\rho}}\right)} \tag{1.7}$$

$\theta_{\rho\mu}$ is not symmetric, and thus is not exactly the energy momentum tensor. It is known as the canonical tensor. From equation (1.6) and the definition (1.4) of P_{μ} it follows that P_{μ} is conserved.

(ii) Conservation of angular momentum is obtained from the invariance under general rotation in the four