

AIN SHAMS UNIVERSITY  
FACULTY OF ENGINEERING

# NONLINEAR ANALYSIS OF CELLULAR STRUCTURES

A Thesis

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By

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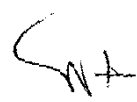


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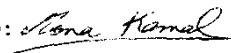
## STATEMENT

This dissertation is submitted to Ain Shams University for the degree of Doctor of Philosophy, in Structural Engineering.

The work included in this thesis was carried out by the auther from November 1990 to January 1994.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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TO MY PARENTS  
MY HUSBAND & SON  
& MY BROTHERS

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The use of reinforced concrete cellular structures has been evolving rapidly during the past decades. This is mainly due to their structural efficiency, and their high resistance to torsional loading. Approximate design methods which were used to analyse box girder bridges under different cases of loading at the service load level, had the advantage of simplicity and the ability of successfully modelling their behaviour under such load levels. But; beyond the service load level, these models did not predict the non-linear behaviour of box girder bridges or the ultimate strength.

In the present research work, a complete investigation has been carried out to study the effect of intermediate diaphragms at different locations on the torsional and distortional behaviour of short, medium and long span cellular structures. Also, the effect of the warping restraint and unrestraint at supports, on the general deformational behaviour of such types of structures is studied.

The present study aims to investigate the effect of changing the diaphragm stiffnesses on the general deformational behaviour of reinforced concrete short, medium and long span cellular structures. Also, the present research work aims to develop a non-linear finite element analysis which analyses box girder bridges throughout their load history. This finite element program takes into consideration the non-linear stress strain curve of concrete, tension stiffening of the cracked concrete elements, strain softening and yielding of reinforcement. Using this program, twenty-four models, subjected to eccentric loading within the serviceability and ultimate stage and up to failure load, were analysed. The varying parameters between these twenty-four cases were the spans, cross-section dimensions, diaphragm locations, and diaphragm thicknesses.

The results of this study were combined with other available information to formulate some recommendations for the designer and researcher, concerning the analysis, design and economy of such type of structures.



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