NUMERICAL SOLUTIONS FOR THE CALCULATIONS OF LAMINAR BOUNDARY LAYER AND POINT OF SEPARATION FOR FLOW PAST CIRCULAR CYLINDERS

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ABSTRACT

Recently, the boundary layer studies have become extremely important. These studies find their applications in the calculation of skin friction drag acting on bodies travelling through fluids, such as the drag on the surface of ships, aeroplane wings and the blades of pumps and turbines.

Boundary layer separation is one of the topics which find great interest from research workers. Separation is usually accompanied by the formation of eddies in the wake of the streamlines.

The equations of motion which describe the flow of a viscous fluid over a solid surface are three non linear partial differential equations, known as Navier-Stokes equations. Till now only approximate solutions are given for these equations except for some few cases. Prandtl (1904) obtained his solution for these equations after studying the order of magnitude of the individual terms and eliminating terms of small order of magnitude. One of these terms that had been neglected by Prandtl and others is the term $\frac{\partial^2 u}{\partial x^2}$ in the equations

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

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for case of two dimensional steady incompressible flow.

The purpose of our work is twofold. First, to study the effect of cavitating conditions on the location of the point of separation. Second, to study the effect of neglecting the term $\frac{\partial^2 u}{\partial x^2}$ on the velocity distribution in the boundary layer.

The study is performed for the case of circular cross-sectional cylinder of 5 cms diameter at Reynold's number of about 9.35×10^4. The pressure distribution around the cylinder is measured in a water cavitation tunnel. This pressure distribution is used for the calculations of the location of the point of separation for different values of the tunnel cavitation number.

A numerical integration routine based on the finite difference method was developed and used to solve the differential equations of motion. The method of solution is programmed in Fortran IV for use on the IBM electronic computer of Ein-Shams University.

The results indicate that the point of separation moves slightly towards the forward stagnation point as the cavitation number increases. Curves for the error arises in the velocity distribution due to neglecting the term $\frac{\partial^2 u}{\partial x^2}$ are also plotted.

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C O N T E N T S

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NOMENCLATURE

x	distance along the curved wall measured from the forward stagnation point.
y	distance along the normal to the wall.
u	velocity component in x direction.
v	velocity component in y direction.
p	pressure.
ν	kinematic viscosity.
ρ	density
$U(x)$	velocity of main stream at the edge of the boundary layer.
ℓ	length.
δ	boundary layer thickness.
$\hat{x}$, $\hat{y}$	non dimensional distances .
$\hat{u}$, $\hat{v}$	non dimensional velocities.
$\hat{p}$	non dimensional pressure
R	Reynold's number
η	dimensionless distance, $\eta = y\sqrt{\frac{a_1}{\nu}}$
a_1, a_3, a_5, a_7	see equation (2.18)
$f_1, f_3, \ldots\ldots$	functions of y given in appendix(B)
δ_1	displacement thickness.
δ_2	momentum thickness.

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Λ	shape factor.
X	see equation (2.39)
Z	see equation (2.40)
ψ	stream function.
$\hat{\psi}$	non dimensional stream function.
U_o	velocity of uniform stream.
$\hat{U}(x)$	non dimensional velocity of the main stream.
Δ	step size in x direction.
h	step size in y direction.
g_5 , h_5	see equation (5.20) and Appendix(B)
g_7, h_7, k_7	see equation (5.21) and appendix(B)
θ	angle measured from horizontal line.
F_j	see equations (5.22).
ϕ_j	see equation (5.26).
δ_j	corrector of velocity, see eq.(5.28).
$P(j)$, $C(J)$, $C'(J)$	see equations (5.30).
$\underline{D}$, $\underline{\Phi}$, $\underline{\Delta}$	matrices
σ	cavitation factor.

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CHAPTER (1)

INTRODUCTION

At the beginning of the present century more attention was given the study of fluid flow in boundary layers. This study finds its application in the calculation of the skin friction drag which acts on a body when moved through a fluid, for example the drag of a ship, of an aeroplane wing and the pump impeller and turbine runner.

Boundary layers have the peculiar property that under certain conditions the flow in the immediate neighbourhood of the solid wall becomes reversed causing the boundary layer to separate from the solid boundary. The separation is accompanied by a more or less pronounced formation of eddies in the wake of the body. The determination of the location of the point of separation is important for the calculation of drag.

The equations of motion which describes the flow of a viscous fluid over a solid surface are three non linear coupled partial differential equations. They are known as Navier-Stokes equations. With the present state of knowledge it is impossible to obtain an exact solution of these equations except for very few cases. Most solutions which can be found in the literature were obtained for approximate versions of Navier-Stokes equations. One of these approximations is the boundary layer equations. Prandtl obtained these equations

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after studying the order of magnitude of the individual terms of Navier-Stokes equations and eliminating terms of small order of magnitude. One of these terms that had been neglected by Prandtl and others is the term $\frac{\partial^2 u}{\partial x^2}$ (see next chapter).

The purpose of this work is dual. First we are going to study the effect of cavitating conditions in the stream of an incompressible fluid on the location of the point of separation. Second, we are going to study the effect of eliminating the term $\frac{\partial^2 u}{\partial x^2}$ on the calculated values of the location of the point of separation. A study of the effect of the same term on the calculated values of the velocity distributions at different sections will be, also obtained.

The two dimensional flow of water around a circular cylinder of 5 cms diameter of the water tunnel of the faculty of engineering, Ein-Shams University is used to obtain the pressure distribution around the cylinder. This pressure distribution is used for the calculations of the location of the point of separation for different values of the tunnel's cavitation number. The static pressure in the main stream and consequently the cavitation number could be controlled using a vacuum pump (see chapter 3).

To study the effect of eliminating the term $\frac{\partial^2 u}{\partial x^2}$ on the location of the separation point, a solution of the boundary

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layer equations is obtained taking this term into consideration. This solution is compared with the solution of the same equations at the same conditions when $\frac{\partial^2 u}{\partial x^2}$ was neglected.

Numerical integration of the boundary layer equations was achieved using a special finite difference scheme in which the truncation error has been minimised. The scheme is programmed in Fortran IV for use on the IBM electronic computer of Ein-Shams University.

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CHAPTER (2)

LITERATURE REVIEW

Prandtl's pioneering work of 1904 started a new era in the field of viscous fluid flow. The literature of this subject has become abundant. In the following we will briefly outline the master works which are relevant to our investigation :

1) In 1904 Prandtl (1) showed how the Navier-Stokes equations could be simplified to yield approximate solutions for problems in which the viscosity is small or the Reynolds number is large. We shall explain these simplifications for the case of two dimensional flow of a fluid with very small viscosity about a cylindrical body of slender cross-section as shown in Fig.(1).

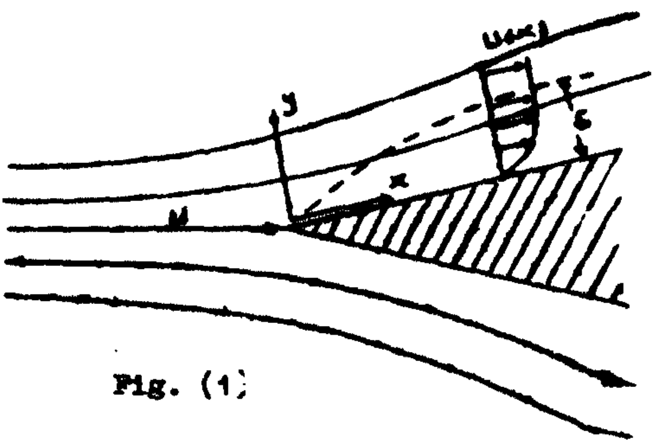

Fig. (1)

The Navier-Stokes equations for steady two dimensional incompressible flow (see reference 2) are

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$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \qquad (2.1)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \qquad (2.2)$$

cintinuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (2.3)

with the boundary conditions

$$u(x, o) = v(x, o) = o, \; u(x, \infty) = U(x)$$

where

- u, v are velocity components in x and y directions.
- $U(x)$ is the velocity outside the boundary layer at distance x .
- p pressure.
- ρ density of fluid.

Dimensionless co-ordinates and variables were used by Prandtl. Let ℓ be some length in the x direction and δ some length in the y direction. Let U be some typical velocity in the x direction and V in the y direction such that

$$\frac{V}{U} = \frac{\delta}{\ell} \qquad (2.4)$$

The following new variables were introduced :

$$\hat{x} = \frac{x}{\ell}, \; \hat{y} = \frac{y}{\delta}, \; \hat{p} = \frac{1}{\rho U^2} p, \; \hat{u} = \frac{u}{U}, \; \hat{v} = \frac{v}{V} \qquad (2.5)$$

Define

$$R = \frac{U \ell}{\nu}, \quad \varepsilon = \frac{\delta}{\ell} \qquad (2.6)$$

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After substituting (2.5) into (2.1), (2.2), (2.3) and making use of (2.4), (2.6), the Navier-Stokes equations becomes

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} = -\frac{\partial \hat{p}}{\partial \hat{x}} + \frac{1}{R}\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \frac{1}{R\,\varepsilon^2}\frac{\partial^2 \hat{u}}{\partial \hat{y}^2} \qquad (2.7)$$

$$\hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} = -\frac{1}{\varepsilon^2}\frac{\partial \hat{p}}{\partial \hat{y}} + \frac{1}{R}\frac{\partial^2 \hat{v}}{\partial \hat{x}^2} + \frac{1}{R\,\varepsilon^2}\frac{\partial^2 \hat{v}}{\partial \hat{y}^2} \qquad (2.8)$$

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \qquad (2.9)$$

Using the dimensional analysis to the original problem to find u as a function of x, y, U, ν, ρ by the Pi theorem. The solution could be expressed in the form

$$\frac{u}{U} = F\left(\frac{y}{x},\ \frac{y}{\sqrt{\nu x/U}}\right) \qquad (2.10)$$

In the dimensionless variables (2.5) this takes the form :

$$\hat{u} = F\left(\varepsilon\frac{\hat{y}}{\hat{x}},\ \varepsilon R^{\frac{1}{2}}\frac{\hat{y}}{\sqrt{\hat{x}}}\right) \qquad (2.11)$$

To find a solution such that $\hat{u}$ depends only upon $\frac{\hat{y}}{\sqrt{\hat{x}}}$.

If we take $\qquad \varepsilon = R^{-\frac{1}{2}} \qquad$ (2.12)

then $\hat{u} = F\left(\frac{1}{R^{\frac{1}{2}}}\frac{\hat{y}}{\hat{x}},\ \frac{\hat{y}}{\sqrt{\hat{x}}}\right)$

and equation (2.7) , (2.8) becomes

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} = -\frac{\partial \hat{p}}{\partial \hat{x}} + \frac{1}{R}\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} \qquad (2.13)$$

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$$\frac{1}{R}\left(\hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}}\right) = -\frac{\partial \hat{p}}{\partial \hat{y}} + \frac{1}{R^2}\frac{\partial^2 \hat{v}}{\partial \hat{x}^2} + \frac{1}{R}\frac{\partial^2 \hat{v}}{\partial \hat{y}^2} \qquad (2.14)$$

Retaining terms through the first order of ε only, one gets

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} = -\frac{\partial \hat{p}}{\partial \hat{x}} + \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} \qquad (2.15)$$

$$\frac{\partial \hat{p}}{\partial \hat{y}} = 0 \qquad (2.16)$$

Continuity equation $\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0$ (2.17)

Thus the equations of motion in its original dimensional form become

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (2.18)$$

$$\frac{\partial p}{\partial y} = 0 \qquad (2.19)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.20)$$

which are known as Prandtl's boundary layer equations.

2) Blasius and Hiemenz (3) constructed a method to give the velocity components u and v at any point in the boundary layer as a power series in x whose coefficients are functions of y. Some of these co-efficients, expressed in non dimensional form, had been evaluated by Hiemenz and improved by Howarth (4). We shall illustrate this method for the case of an obstacle which is symmetrical

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