ON THE RELIABILITY OF THE ONE UNIT STAND-BY WITH REPAIR AND MAINTENANCE

A THESIS

Submitted in Partial Fulfilment
of the Requirements for the Award of the
M.Sc. Degree

By

ADEL ABDEL RAHMAN ABDEL HAFEEZ
B.Sc. Hons. 1973

Supervised by

Professor ABD EL-AZIM ANIS
Professor of Mathematical Statistics
Departement of Pure Mathematics

Dr. YOUSSEF NASR EDDI[illegible]
Senior Expert

Faculty of Science
Ain Shams University

Insitute of National
Planning

1976

M.Sc. Courses

Studied by the Author (1973-1974)

1.	Theory of probability	2 hours weekly
2.	Theory of storage	2 hours weekly
3.	Stochastic processes	2 hours weekly
4.	Design of experiments	2 hours weekly
5.	Nonparametric statistics	2 hours weekly

بسم الله الرحمن الرحيم

ACKNOWLEDGEMENT

I should like to express my sincerest thanks to Professor A.A. Anis, professor of Mathematical Statistics, Departement of Pure Mathematics, Ain Shams University, for his constant encouragement and for reading the manuscript.

I wish to acknowledge my deepest gratitude and thankfulness to Dr. Yousif Nasr Eldin, senior expert, Institute of National Planning, for suggesting the topic of the thesis, for his kind supervision, and for his invaluable help during the preparation of the thesis.

CONTENTS

Page

INTRODUCTION 1

CHAPTER I

THE EFFECT OF REDUNDANT SYSTEM WITH AND WITHOUT REPAIR

I-1. Introduction 2

I-2. Reliability function, Failure function and Failure rate 5

I-3. Mean life of a single device 10

I-4. Methods for improving reliability 11

I-5. When Preventive maintenance is advisiable? .. 12

CHAPTER II

ON SYSTEMS WITH REPAIR AND PREVENTIVE MAINTENANCE

II-1. Introduction.............................. 39

II-2. Construction of the system 40

II-3. The maximization of the mean life time... 71

II-4. A system of n operative units and one stand -by 75

CHAPTER III

THE DUPLICATION SYSTEM WITH REPAIR AND PREVENTIVE MAINTENANCE UNDER POLICY I

III-1. Introduction 88

CONTENTS

Page

III-2. The survival function 88

III-3. Comparison of the mean times of the system failure 106

III-4. Conclusion 114

REFERENCES.................................... 116

ARABIC SUMMARY

-oOo-

INTRODUCTION

The mathematical theory of reliability has grown out of the demands of modern technology and particularly out of the experiences in World War II with complex military systems.

The complex devices are utilized for military and scientific purposes. To increase the efficiency of these devices we use the theory of reliability.

Many definitions for reliability have been given and varied among different writers. But there is a definition of reliability given by RETMA [6] ; this definition is now accepted by most reliability authorities, and hence is considered to be standard. This definition states that " Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered ".

The most common measures of reliability are failure rate, probability of survival and the mean time between failures. But the time that is taken in repair or maintenance processes, the number of failures of an equipment in certain time interval, the length of life of an equipment, ... etc. are all events which vary in more or less random ways. The exact behaviour of these random variables is uncertain, thus the probability theory is one of reliability fields of study.

There are various techniques and methods for improving reliability, the most basic method is through systems engineered by mature design.

The useful technique to give a high degree of reliability is redundancy. There are two basic types of redundancy, namely, stand-by redundancy and parallel redundancy.

Redundancy has proved to be a significant technique in achieving a high level of reliability, but this type of arrangement is not necessary if one component is of sufficient reliability because the cost factors of redundancy violate simplicity and human error.

In this thesis we aim to improve a stand-by complex system reliability. The problems such as cost problems, repairman problems, etc., are not considered. We take only the mean life time as an absolute measure for the system reliability.

Maintainability is also used to obtain a high level of reliability. There are many definitions for maintainability, a definition which is simple and uses quantitative terms is more desirable. This quantitative definition has been presented by Goldman [6], it states that " maintainability is the probability that a device will be restored to operational effectiveness within a given period of time when the maintenance action is performed in accordance with prescribed procedures ".

There are two types of maintainability, namely, corrective maintenance and preventive maintenance with which we are mainly concerned in this thesis under two policies, policy I , the operative unit undergoes inspection when the inspection time comes, only if the other unit is in stand-by state. Thus no inspection takes place for the operative unit if the other unit is under repair or inspection; and policy II, the operative unit undergoes inspection when the inspection time comes regardless of the state of the other unit.

The theris is divided into three chapters:

Chapter (I) is concerned with the effect of redundancy with and without repair (corrective maintenance). In this chapter we give a survey of the problems under certain assumptions, the Laplace transforms of the distributions of the failure time of the system, the mean life time and we answer the question: when is preventive maintenance advisable?

In the second chapter we deal with preventive maintenance under policy (II), we discuss the duplex system in general and under special cases, and we give for the first time the system of equations of a system which consists of n operative units and one stand-by with repair and preventive maintenance under some assumptions; we discuss some special cases, and we give a theorem which states that " The mean life time of the system with repair and maintenance is less than the mean life time of the system with repair only if the failure time distribution is exponential and the time to inspection distribution;

$$U(x) = \begin{cases} 0 & x < T \\ 1 & x \geq T \end{cases}$$

In the third chapter we deal with the duplication system under policy (I) , in general and under special cases. The results of this chapter are new.

CHAPTER (I)

THE EFFECT OF REDUNDANT SYSTEM WITH AND WITHOUT REPAIR.

I-1) Introduction:

Nowdays, in which complex military systems and large electronic computers are used for military and scientific purposes, a high degree of system reli - ability is an absolute necessity. This necessity has also grown out of the continuous demands of modern technologies and the requirment of a high accuracy level of equipment function for weapons and other important systems. Unreliability has consequences in cost, time wasted, psychological effect of incon- venience and national security. Reliability then is an important research area; it is every body's business. Certainly, it is desirable to construct a system which is 100% reliable. Unfortunately, we cannot expect such a high level of reliability when the reliability factors are human beings, understand- ing of the problem, system of communications and requirments of experience.

The most common measures of reliability are failure rate, probability of survival or mean time between failures. But, the length of life of a device or system, the time that is taken in repair or maintenance processes, the number of failures of an equipment in a certain time interval, the number of units in a queue waiting for a service,... etc are all events which vary in more or less random ways.

The calculation or determination of the exact behaviour of these random variables is not possible, since uncertainty due to randomness and due to our ignorance of the true state of the system is obvious. Probability is a measure of uncertainty. Therefore, the probability theory is one of the most important tools for solving reliability problems. Consequently, reliability theory is mainly concerned with probabilities, mean values, probability distributions ... etc.

On the other hand , reliability problems have a structure of their own, and thus stimulated the development of new areas in probability theory itself.

Generally speaking, reliability theory is directed towards the solution of problems in, optimizing the probability of mean life, decisions concerning maintenance policies to be followed, or probabilistic models.

Many definitions for reliability have been given and varied among different writers. The RETMA definition of reliability states" Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered ". This definition is now accepted by most reliability authorities and hence is considered to be standard. We can see that the definition is a probability statement with qualifying conditions which are open to interpretation according to the numerous particular situations. The quality of reliability can be associated with a number, in most cases, by processes of estimation. Mathematical statistics, is used to do this by relating observed past events to future similar events by means of probability statements. These reliability estimates can be considered as though they were exact measurements and therefore a decision may be made on their adequacy

I-2) Reliability function, Failure function and Failure rate:

Reliability has been defined in terms of probability and as a function of time; therefore, when discussing reliability we always refer to a period of time, usually, the interval (o,t) . The reliability distribution function R(t), known as survival function, is a specified function which describes the adequacy state of a system after certain period of time.

If $X(t) = (X_1(t), X_2(t), \ldots, X_n(t))$ is a vector-valued random variable which describes a system state at time t, and, if $x_1, x_2, \ldots, x_n$ are the limits which define the event success, then, R(t) is given by the probability that

$$X_1(t) \geq x_1, \; X_2(t) \geq x_2, \ldots, X_n(t) \geq x_n$$

i.e.,

$$R(t) = \text{Prob}\left\{ X_1(t) \geq x_1, X_2(t) \geq x_2, \ldots, X_n(t) \geq x_n \right\} \qquad \ldots (I-2.1)$$