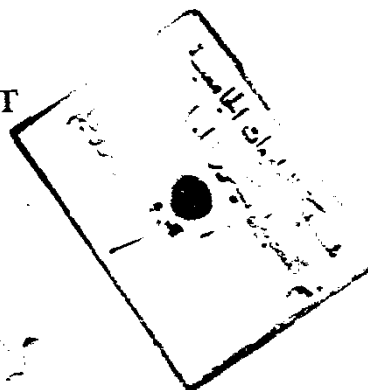


ON THE FUNCTOR GENERATED BY A COHOMOLOGY

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SUMMARY

SUMMARY

It should be kept in mind that homology and cohomology theories are over 90 years old, and the entire subject has been worked over and added to by some of the most ingenious and imaginative mathematicians of the world. The two main kinds of homology and cohomology theories are the singular theory, [19], and the Čech-Alexander-Spanier type theory, [11], [33]. Although the two theories give the same results on topological spaces which are locally nice, it is well known that on general spaces the Čech-Alexander-Spanier type cohomology theory has several technical advantages over the singular theory. For example, M. Barratt and J. Milnor, [4], have given a simple example of a compact subset A of \mathbb{R}^3 such that the singular cohomology groups with rational coefficients, $H^n(A, \mathbb{Q})$, are nonzero for infinitely many values of n .

Indeed this is one of the most striking differences between singular homology or cohomology, and the Čech-Alexander-Spanier type of homology or cohomology. In spite of these advantages, the definition and development of the homology theory associated with Čech-Alexander-Spanier cohomology has heretofore seemed rather involved and complicated.

Massey, [23], [24], has given a clear and simple systematic exposition of the Čech-Alexander-Spanier type of cohomology theory and its associated homology theory, which is sometimes referred to

