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A STUDY ON THE MOTION OF MULTICOMPONENT MEDIA IN PIPES

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## NOTATIONS

$A_1$	average thickness of the first phase
$A_2$	average thickness of the second phase
$A$	amplitude of the interfacial wave
$B$	magnitude of variation in the cross area of flow
$E$	complex valued function of $y$
$E^i$	$i$ th component of the electric field
$F$	complex valued function of $y$
$F_e$	electromagnetic force
$F_g$	non electric force
$f(x)$	real valued function of $x$
$2H$	interface curvature
$H_0$	magnetic field intensity
$H^i$	$i$ th component of the magnetic field
$i$	complex variable
$J^i$	$i$ th component of the electric current density
$l$	wave length of the periodic solid wall
$m$	$A_1 / A_2$
$P$	pressure
$P_a$	ambient pressure
PDES	partial differential equations
$Q$	axial flow rate per unit length
$q$	$Q^{(2)} / Q^{(1)}$
$R$	$Q/\nu$ , Reynolds number
$t$	time
$T$	stress tensor
$T_{na}$	normalized stress tensor
$T_{nn}$	normalized stress tensor
$u$	$x$ -direction velocity

$v$	y-direction velocity
$w$	z-direction velocity
$x$	Cartesian coordinate
$z$	Cartesian coordinate
$Z_0$	Position of the solid wall
$Z_1$	Position of the fluid-fluid interface
Greek symbols	
$\alpha$	$\mu^{(2)} / \mu^{(1)}$ ratio of viscosities
$\beta$	$\sigma \mu H_0$ magnetic parameter
$\beta_1$	$n\beta$ magnetic parameter
$\gamma$	Phase shift between fluid-solid and fluid-fluid interfaces
$\varepsilon$	$B/A_2$
$\theta$	$\rho^{(2)} / \rho^{(1)}$
$\Lambda$	$\mu Q / \sigma_1 A_1$ , capillary number
$\lambda$	$2 \pi A_2 / l$
$\mu$	viscosity
$\mu_e$	magnetic permeability
$\nu$	Kinematic viscosity
$\rho$	density
$\rho^{(1)}$	density for first phase
$\rho^{(2)}$	density for second phase
$\rho_e$	electric charge density
$\sigma_1$	surface tension
$\sigma$	conductivity of the medium
$\Omega$	force potential of the field in which flow occurs
$\psi$	stream function
$\Delta$	Laplacian operator

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## ENGLISH SUMMARY

## SUMMARY

This study deals with some problems suspended with the motion of multicomponent media in pipes .

The results of this thesis are very important from both a conceptual and an applicative point of view such that , in chemical industries , Nuclear reactors , Petrol industry and in some various industrial devices .

This thesis consists of five chapters :

Chapter one includes an introductory review and a general review of research works related to the proposed point under consideration is presented .

Chapter two contains the problem " Electrical conducting film flow down an inclined periodic plate " , the solution of the problem depends on the analysis of the flow of one fluid through a periodic plate , a regular perturbation analysis is performed to evaluate the velocity field of the one fluid flow and its interfacial shape , we found that the amplitude and the phase shift of the stream function depends on the parameters ( $\beta, \lambda, A$  and  $D$ ) . The main results obtained in this chapter are summarized as evaluations of the flow of one fluid through a periodic plate show that the magnetic field increases the amplitude ( $A$ ) of the stream function and decreases the phase shift ( $\gamma$ ) of the stream function . There exist some results

depends on the parameters ( $\beta, \lambda, \Lambda$  and  $D$ ) .

Chapter three contains the problem " Slow two-phase flow through a sinusoidal channel with porous wall " , the solution of the flow depends on the parameters ;  $\beta$  : the magnetic field parameter ;  $\alpha$  : the ratio of viscosities ;  $\Lambda$  : the capillary number ;  $\lambda$  : the geometrical parameter ;  $m$  : the ratio between average thickness ;  $q$  : the relation between flow rates and  $V$  : the suction parameter . A regular perturbation analysis is performed to evaluate the velocity profile . The main results obtained in this chapter are summarized as follows , the amplitude of the stream function ( $A$ ) decreases as the parameter of suction ( $V$ ) is increased , the phase shift of the stream function increases as the parameter of suction ( $V$ ) is increased . There is also some results on the parameters ( $\lambda, \alpha, \Lambda, q$  and  $m$ ) .

Chapter four contains the problem " Two-phase flow in a sinusoidal channel when one of the phases is electrically conducting " . The solution of the problem depends on the analysis of the two-phase flow through a sinusoidal channel , a regular perturbation analysis is performed to evaluate the velocity profile of the two-phase flow and its interfacial shape.

we found that the solution of the problem depends on the parameters ( $\beta, \alpha, \lambda, \Lambda, m$  and  $q$ ) . The main results obtained in this chapter are summarized as follows the magnetic field increases the amplitude ( $A$ ) of the stream function and decreases the phase shift of the stream function ( $\gamma$ ) . The magnetic field increases the shear stress . The phase shift presents a peak in the solid

fluid interface and presents a bottom in the fluid fluid interface . There is also some results on the parameters ( $m, q, \alpha, \lambda, A$  and  $\beta$ ) .

Chapter five contains the problem " flow of an electrically conducting two-phase flow through a symmetric sinusoidal channel when one of the two-phase is electrically conducting  $n$ -times than the other phase . The solution of the problem depends on the parameters ( $\beta, \beta_1, \alpha, A, \lambda, m$  and  $q$ ) . A regular perturbation analysis is performed to evaluate the velocity profile of the two phase flow and its interfacial shape . The main results obtained in this chapter are summarized as follows , the amplitude of the stream function ( $A$ ) do not change when one of the two phases is conducting  $n$ -times than the other phase and the phase shift of the stream function ( $\gamma$ ) changes slightly . The magnetic field increases the shearing stress of the flow field , since the shearing stress due to the magnetic field is proportional to the magnetic field  $H$  which will be zero on the wall . The shear stress increases as the magnetic field is increased .

**INTRODUCTION  
AND  
AIM OF WORK**

## CHAPTER I

## CHAPTER I

### INTRODUCTION

#### I.1. INTRODUCTION :

Magnetohydrodynamics (MHD) differs from ordinary hydrodynamics in that the fluid is electrically conducting . It is not magnetic , it affects a magnetic field not by its mere presence but only by virtue of electric currents flowing in it .

Multiphase systems consist of a fluid phase or fluid medium and a particulate phase of any number of chemical components . When the fluid medium is a liquid , the particulate phase may consist of solid particles , gas bubbles , or liquid droplets .

The dynamics of multiphase systems include momentum , energy , mass , and charge transfers between the phases whether or not the process is influenced by the presence of a potential field . There are many multiphase systems among engineering equipment and process .

Some of this systems may be obtained as follows :

- a. Gas solid particle systems ; such as dust collectors , cosmic dusts , and nuclear fallout problems .
- b. Gas-liquid droplet systems ; such as atomizers , air pollution and combustors .

- c. Liquid-gas bubble systems ; such as absorbers , air lift pump and aeration .
- d. Liquid-liquid droplet systems ; such as extraction .
- e. Liquid-solid particle systems ; such as sedimentation .

Studies of dynamics of multiphase systems have followed two methods of approach [3] :

1- Treating the dynamics of single particles and then trying to extend to a multiphase particle system in an analogous manner as in molecular (kinetic) theory .

2- Modifying the continuum mechanics of single-phase fluids in such a way as to account for the presence of particles .

A study of the phenomena occurring in the motion of multiphase media is essential for the solution of many theoretical and engineering problems : the theory of wave propagation in water-saturated ground and dust-saturated air , the theory of fine grinding , the theory of mass entrainment from the surface of a body by a high-parameter flow , the theory of the flow around porous bodies , the theory of the motion of powder gases containing unburned particles , the perfection of water and pneumatic transport , petroleum output and petroleum refining when there is foreign matter in the petroleum , etc. In all three cases the moving material is a mixture of two or more media , i.e. phases .