

AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING

STRUCTURAL ANALYSIS FOR SOME TYPES OF
THIN RECTANGULAR PLATES
BY
USING FINITE DIFFERENCE METHOD

By

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THESIS

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STATEMENT

This dissertation is submitted to Ain Shams University for the degree of Master of Science in Civil Engineering.

The work included in this thesis was carried out by the author in the department of Civil Engineering, Ain Shams University, from December 1984 to February 1991.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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ABSTRACT OF THE M.Sc. THESIS

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Abstract:

Often encountered in practice are, conventional types of thin rectangular plates (simply supported plate, flat slab,...etc) that are used as floors in buildings for common purposes, rectangular ribbed plates stiffened by orthogonal ribs, and the rectangular plate having a rectangular hole with edges parallel to the plate edges that are, usually, used for large areas and heavy concentrated loads in industrial building.

The previous methods that produce the exact solutions of the above mentioned cases of plates, most of them, except the finite element analysis method, is valid only for specified loading and boundary conditions at the plate edges.

Herein, a mathematical modeling is prepared by using the finite difference analysis and successive over relaxation

method: it is used to analyze the behavior of the previous types of plates which are subjected to uniformly distributed loading, or concentrated loading at some points along the plate surface.

The results of this model for some types of rectangular plates (solid plate, ribbed plate or having rectangular hole) with different boundary conditions and different types of vertical loading are confirmed by comparing them with the results of the previous works. And it is used to make the parametric study for simply supported orthogonal ribbed plate to investigate the effect of torsional rigidity and stiffness rigidity of ribs on the resulted internal forces of plates due to transversely uniform loading, also it is used to study the distribution of the internal forces within the plate surface around the rectangular hole due to vertical loading.

key words:

Finite difference, Successive Over Relaxation, Relaxation parameter, Ribbed plates, Stiffness rigidity ratio, Torsional rigidity ratio, Rectangular hole, Boundary conditions.

CHAPTER (1)
INTRODUCTION

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INTRODUCTION

(1.1) Introduction:

Thin rectangular plates (simply supported plate, continuous plate, flat slab, ...etc), are often used as floors in buildings for common purposes. For large areas when the presence of interior columns is not desirable as in industrial building, garages, conference halls,etc, or for heavy concentrated loads, the plate is stiffened by ribs either in one direction or in two directions to decrease the dead load of the structure and increase the bending stiffness of the plate. Also we may have a rectangular hole as in core or in industrial buildings.

The structural analysis of thin rectangular plate element started in 1811 by Lagrange; and since then many researches to study different types of thin plates were made by using different techniques either mathematical closed form solutions or numerical solutions. The mathematical closed form solutions, are sound only for specified loading and boundary conditions at the plate edges. The numerical methods, on the other hand, are more versatile and applicable to a large variety of plate problems.

Herein, a mathematical modeling is prepared by using the finite difference technique and Successive Over Relaxation Method, (SORM), and it is used to analyze the behavior of rectangular plate either of conventional types (solid, continuous, flat slab, ...etc), ribbed plate with different boundary conditions or having a rectangular hole within its boundaries, and subjected to uniformly distributed loading, nonuniform distributed loading or concentrated loading at some points along the plate surface.

The results of this model for some types of rectangular plates (conventional plates, ribbed plates or having a rectangular hole) with different boundary conditions and different types of vertical loading are confirmed by comparing them with the results of the previous works. This model is then used to make a parametric study for simply supported orthogonal ribbed plates to investigate the effect of torsional rigidity and stiffness of ribs on the resulted internal forces due to transversely uniform loading; also it is used to study the distribution of the internal forces within the plate surface around the rectangular hole due to vertical loading.

The present method is considered a direct one to produce the distribution of the internal forces (deflections and moments) for any configuration of a thin rectangular plate either, conventional type, stiffened or having a rectangular hole subjected to different conditions, while any one of the previous exact solution methods does not have this advantage.

(1.2) Objects of the dissertation

The object of this dissertation is to construct a new mathematical model by using finite difference technique and the successive over relaxation method to perform the following tasks:

- (1) Study the structural behavior of a thin rectangular plate subjected to a uniform, nonuniform or a concentrated load with different boundary conditions.
- (2) Investigate the effect of torsional rigidity and stiffness rigidity of ribs of simply supported rectangular ribbed plate subjected to transverse uniformly distributed loading on the plate internal

forces; also study the effect of different boundary conditions, number of orthogonal ribs and types of vertical loading on the internal forces of the plate.

- (3) Determine the effect of existence of rectangular hole through thin rectangular plate -(with the condition that the edges of the hole must be parallel to the edges of the plate), subjected to transverse uniform or concentrated loading- on the internal forces of plate with different boundary conditions.

The analysis represented here applies to the elastic thin rectangular plates using the small deflection theory of the plate bending (Kirchoff hypotheses), as given in section (2.2).

(1.3) Layout of the study.

This study consists of six chapters labelled from (chapter 1) to (chapter 6); each chapter is concerned with a specific task as follows:

Chapter 1, gives the introduction to the work and the objectives of the dissertation.

Chapter 2, reviews the derivation of the partial differential equations of equilibrium for a thin rectangular plate element and for ribbed plate element. It also gives the differential equations for different boundary conditions.

Chapter 3, reviews derivation of the various difference equations either for equilibrium of plate element and ribbed plate element or for different boundary conditions from the first principles based on the averaged first central difference. It then gives the construction of the mathematical

model. Also, this chapter gives a comparison between the results of this model by using the Relaxation method (SORM), and the exact solution in the form of Fairer's series, for some square plates with different conditions and types of vertical load.

Chapter 4, Investigates the solution of the stiffened rectangular plate with orthogonal ribs by using this model. Also, this chapter contains the results of this model for three different cases of study: one for the effect of different boundary conditions, uniform load and concentrated load, second for the effect of increasing the stiffness of ribs, and the third to study the effect of increasing the torsional rigidity constant of ribs.

Chapter 5, is concerned with the study of the distribution of internal forces and deformations around the hole for a thin rectangular plate having a rectangular hole with edges parallel to the plate boundaries. The hole's edges may be simply supported, built-in or free. The edges of the plate may be simply supported or built-in. The plate is loaded with either a uniform load or a concentrated load.

Chapter 6, contains the conclusions for the types of thin rectangular plates that has been studied in this dissertation.