

**LIMIT STATE DESIGN AND ANALYSIS OF
BIAXIALLY LOADED RECTANGULAR
REINFORCED CONCRETE SHORT COLUMNS**

By

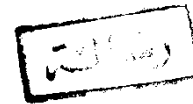
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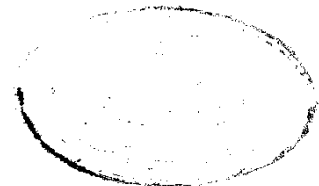
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*To My Parents
and My Lovely Sister*



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rectangular reinforced concrete short columns "**

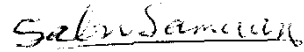
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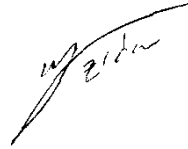
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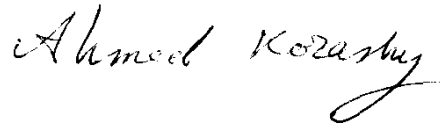
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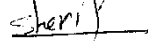
STATEMENT

This Thesis is submitted to Ain Shams University for the degree of Master of science in Structural Engineering.

The work included in this thesis was carried out by the author in the department of structural Engineering, Ain Shams University, from December 1990 till October 1993.

No part of this thesis has been submitted for a degree or a qualification at any other university or institute.

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Abstract : The research offers an approximate method for the analysis and design of biaxially loaded rectangular reinforced concrete short columns. The method applies the assumptions of the limit state design of reinforced concrete members. The method is simple and efficient; moreover, the depth of the column could be estimated. The concept of this method is to construct a modifier failure surface in a table form. The modifier failure surface is defined by radius, horizontal and vertical angles. The method assumes that the horizontal angle is constant for a certain range that encompasses the practical range of ratios of eccentricities to the columns dimensions. All the values in the tables represent the radius length of the modifier failure surface at a certain vertical angle and reinforcement ratio. The research presents two computer programmes to analyze biaxially loaded columns based on earlier research works. The research, also, presents a comparison between the proposed method and the Egyptian code method for design of biaxially loaded columns.

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List of Notations

The following symbols have been adapted for use in this thesis. Other symbols not listed below are defined where they first appear.

Chapter (1)

Nil

Chapter (2)

a and b	:	entire dimension for cross section
β	:	Coefficient
M_e	:	Equivalent moment (method of AAS - Jakobson)
c	:	Parameter (method of AAS - Jakobsen)
t_x and t_y	:	Dimension of cross section.
p_c and p_s	:	Carrying capacities of concrete and steel.
M_{ox}	:	Maximum moment capacity when $e_y = 0$.
M_{oy}	:	Maximum moment capacity when $e_x = 0$.
α and β	:	Parameters used by Bresler.
β_x , β_y and α_n	:	Parameters used by (C.P.110)

Chapter (3)

K_x and K_y	:	Two axes of the modifier failure surface in horizontal plane.
K	:	Vertical axis of the modifier failure surface.
b	:	Width of section.
t	:	Height of section.
γ_c and γ_s	:	Strength reduction factors for concrete and steel, respectively.

A_{sx} and A_{sy}	: Number of bar reinforcement at X and Y directions, respectively.
P	: Nominal axial load strength, ton.
M_x and M_y	: Nominal moments strength about x and y axes, mt.
x and y	: Axes of C.L. of the cross section.
A_s	: Area of steel
μ	: Reinforcement ratio.
ρ	: Parameter.
R	: Radius of the modifier failure surface at a certain point.
r	: Radius of constant load contour which passes through the same point.
Level	: Vertical angle of that point.
θ	: Horizontal angle of that point.
e_x and e_y	: Eccentricities along x and y axes, respectively.
F_{cu}	: Characteristic strength of concrete.
F_y	: Yield or proof strength of steel.
e	: eccentricity along x or y axis.
X_n and Y_n	: The intersection of neutral axis with X and Y axes, respectively.
E_s	: Modulus of elasticity of steel.; $E_s = 2000 \text{ t / cm}^2$.
strain 1	: Maximum strain.; strain1 = 0.003.
strain Y	: Strain of steel at yielding
S	: Segment number, s = 1 to 70.
X_s	: Intersection of C.L. of a segment (s) with X-axis.
Y_s	: Intersection of C.L. of a segment (s) with Y-axis.
h	: width of any segment.
g	: Indication to a big segment.
g_B	: Indication to a bottom segment.

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g_T	: Indication to a top segment.
p, m_x and m_y	: The contribution in P, M_x and M_y of one segment.
X_g and Y_g	: Coordinates of c.g. of a big segment.
X_B and Y_B	: Coordinates of c.g. of a bottom segment.
X_T and Y_T	: Coordinates of c.g. of a top segment.
X_a and Y_b	: Intersection of line passes by points of strain = 0.002.
m	: Number of trials.
α	: Skew angle of neutral axis.
M_{rx}	: Required moment capacity about x-axis.
M_{ry}	: Required moment capacity about y-axis.
P_r	: Required axial load capacity.
ζ	: Vertical angle for the interaction.
C	: Distance from extreme compression fiber to neutral axis.
P_x	: Load capacity of a section with e_x only.
f_y	: load capacity of a section with e_y only.
P_o	: axial load capacity ($e_x = e_y = \text{zero}$).

Chapter (4)

Nil

Chapter (5)

Nil

List of Tables

Chapter (1)	Nil
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