# AIN-SHAMS UNIVESITY FACULTY OF ENGINEERING

BEHAVIOUR OF REINFORCED CONCRETE ELEMENTS
UNDER VARIOUS LOADING CONDITIONS

(EFFECT OF REVERSED FLEXURE)

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#### THESIS

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## APPROVAL SHEET

# BEHAVIOUR OF REINFORCED CONCRETE ELEMENTS UNDER VARIOUS LOADING CONDITIONS (EFFECT OF REVERSED FLEXURE)

Ph.D. Thesis

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#### STATEMENT

This dissertation is submitted to The Faculty of Engineering, Ain-Shams University for the degree of Doctor Of Philosophy in Civil (Structural) Engineering.

The work included in this thesis was carried out by the author through a channel system between "the Department of Structural Engineering, Ain-Shams University" in Egypt and "Centre Experimental De Recherches Et D'Etudes Du Batiment Et Des Travaux Publics - C.E.B.T.P." in France.

No part of this thesis has been submitted for a degree of qualification at any other university or institution.

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To my father ...

AIN-SHAMS UNIVERSITY
FACULTY OF ENGINEERING
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Abstract of the Ph.D. thesis submitted by:

Eng. Fatma Ahmed Shaker.

Thesis Title: Behaviour Of Reinforced Concrete Members
Under Various Loading Conditions
(Effect Of Reversed Flexure)

<u>Supervisors:</u> Prof.Dr. Abdel-Hady Hosny & Prof.Dr. Jacques Trinh. <u>Registration Date:</u> 15/2/1982 Examination Date: / /1990

#### ABSTRACT:

This thesis presents experimental and theoretical investigations on the influence of cyclic reversed eccentric compressive loads on R.C. members. The experimental research includes 2 series of test specimens. Different parameters are considered. The results include load-deflection and load-strain diagrams for both concrete and steel as well as cracking patterns.

basic governing equations and formulation of a non-linear finite element program adapted for microcomputer applications are given. This program performs two-dimensional analysis of R.C. structures. It simulates the following features of concrete behaviour: cracking - crack opening and closure - nonlinear behaviour under compressive stresses - effect of biaxial state of stresses - strain softening postcrushing - behaviour under loading and unloading conditions. The program idealizes the steel bars and bond between concrete and steel by elasto-plastic truss elements. Checks proving the validity and precision of calculations are given, then the program is used in simulating the experimental work. The sufficiently good agreement between theoretical and experimental results proved the reliability of the utilized finite-element modeling and the method of analysis. Interaction envelopes of R.C. members under monotonic loading "Conventional conditions Envelopes" are discussed criticized. then 2 other types of envelopes are proposed. are Yielding Limit Envelopes and Elasto-Plastic Envelopes.

a parametric study concerning the effect of cyclic Finally reversed eccentric compressive loads on the envelopes presented is through non-linear finite-element analyses. The considered parameters are: concrete grade, steel yield stress, reinforcement ratio, cross-sectional total depth to breadth ratio and number of cycles. The interaction envelopes after cyclic loading are compared with the above mentioned 3 types of monotonic loading interaction envelopes.

#### Key Words:

Reinforced concrete-cyclic loading-reversed loading-eccentric compression-finite element-bond-deflection-strains-stresses-cracking-steel yield-interaction envelopes.

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#### **NOTATIONS**

#### I. FINITE-ELEMENT NOTATIONS:

- al & a2 Factors defining the bi-axial strength envelope of concrete.
- [B] Strain-displacement transformation matrix.
- [D] Matrix relating between strains and stresses.
- fe' Concrete characteristic strength that is taken in this work equal to the crushing strength (oc).
- Eo Initial tangent modulus of elasticity.
- Ecomp Modulus of elasticity corresponding to Ecomp and Geomp.
- Ecurr Modified current modulus of elasticity during calculations to take into account the effect of biaxial state of stresses on the uniaxial stress-strain curve of concrete.
- Ep1 & Ep2 Modulii of elasticity in the direction of principal axes in a plane stress orthotropic concrete element.
- {Ft} Vector of nodal forces at solution step (t).
- [J] Jacobian matrix relating between the cartesian and natural co-ordinate systems.
- det J Determinate of the Jacobian matrix.
- [K] Structural stiffness matrix.
- [Ke] Stiffness matrix at a finite-element (e).
- [Kt] Stiffness matrix at solution step (t).
- kij A component in the stiffness matrix of a finite
  element (e).
- [M] Structural mass matrix.
- Mm Reduction factor for cracked concrete stiffness in the direction normal to the crack.

Ms Reduction factor for the shear stiffness of cracked concrete in the direction of the crack. the direction perpendicular to the crack. [N]matrix of shape interpolation functions. Total number of nodes within an element. P r & s Natural co-ordinates system. rt. & sk Natural co-ordinates of an integration point. {Rt} Vector of applied loads at solution step (t). th Thickness of a plane stress finite element. {U} Vector Of nodal displacements. [ Ut ]i Vector Of incremental displacements at the solution step (t) and at iteration (i). Local displacement at node (i) in the direction of ui X-axis. {0} Vector of nodal point acceleration. ¥ Local displacement at node (i) in the direction of Y-axis. WL & Wk Weighting factors in the numerical integration process. X-cartesian co-ordinate at node i. Xi Y-cartesian co-ordinate at node i. Уi **{δ}** Vector containing displacement components. Poisson's ratio. τ Angular plane rotation at a point. TX Y {€} Vector containing strain components. €c Crushing strain of concrete. €e ' Modified crushing strain of concrete under bi-axial compressive state of stresses. €сощр Compressive strain. €ŧ Cracking strain of concrete. Eten Tensile strain in concrete. Ultimate strain of concrete. €u €u' Modified ultimate strain of concrete under bi-axial

Strain at a point in the direction of X-axis.

compressive state of stresses.

 $\in_{\mathbf{x}}$