

AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING

NONLINEAR ANALYSIS AND DESIGN
OF REINFORCED CONCRETE
PLATES AND SHELLS

A Thesis
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By

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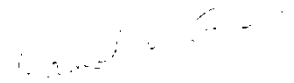
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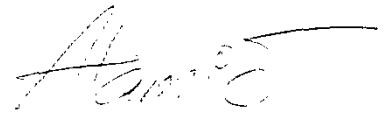
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STATEMENT

This dissertation is submitted to Ain Shams University for the degree of Doctor of Philosophy, in Structural Engineering.

The work included in this thesis was carried out by the author from June 1988 to June 1994.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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To

My Wife

My Children

My Mother

I dedicate this work to
all those who have contributed to my education

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ABSTRACT

It is unclear whether reinforced concrete flat-plate structures possess sufficient lateral capacity to resist the lateral forces that can be expected during a strong wind or an earthquake. Most experimental studies on flat slab structures have been concentrated on individual element behavior. Some researchers studied connections with edge columns subjected to shear-moment transfer, while others studied interior slab-column connections. This type of investigations can not predict the actual behavior of flat plate structures under vertical and increasing lateral loading. Therefore experimental study is needed to cover this part.

During recent years much research has been devoted to the development of nonlinear numerical methods of analysis for reinforced concrete structures. At present , several computer programs are available for the study of the nonlinear response of

reinforced concrete slabs. None of the available programs account for all critical factors affecting the nonlinear analysis of reinforced concrete structures. Therefore analytical study is needed to investigate these factors.

The objective of this study is to experimentally and analytically investigate the behavior of reinforced concrete flat-plate structures under simultaneous uniformly distributed vertical load and increasing lateral load up to failure with the aid of direct 1/4 and 1/10 scale models. The dimensions and loading of the models were chosen to simulate the behavior of actual prototype structures and are used to study the column-slab connections and the overall 3-dimensional behavior of the structures.

Due to the prohibitive cost of full-scale testing of reinforced concrete structures, potential problems of workmanship and capacity limitation of loading equipment a more economical method utilizing the direct modeling technique is proposed as a replacement to full-scale testing. The technique and methodology used in modeling the reinforced concrete specimens at 1/4 and 1/10 scale are presented and discussed. The geometrical, physical and mechanical properties of all model component materials (concrete and steel reinforcement) are investigated experimentally.

The present experimental study focuses on the behavior of the reinforced concrete building frames subjected to constant vertical load and monotonically increasing lateral load and reversed cyclic loading representing wind or earthquake actions using small-scale direct modeling techniques. This would provide to have a better understanding of the behavioral characteristics of the reinforced concrete building frames under combined vertical and lateral loads.

A total of four specimens were constructed and tested at Drexel University Laboratory. These specimens are:

1. Fixed plane frame one bay, one storey using 1/4 scale model was tested under constant concentrated vertical load at the middle and increasing monotonic lateral load up to failure.
2. One storey, one panel, two way slab on four beams supported on four columns using 1/4 scale model, was tested under constant uniform vertical load (using a waffle tree and hydraulic jack) and increasing monotonic lateral load up to failure.
3. One storey, one panel, flat plate on four columns, using 1/4 scale model, was tested under constant vertical load (using waffle tree and hydraulic jack) and increasing monotonic lateral load up to failure.
4. Three storey, one bay by three bays flat plate on eight columns using 1/10 scale model, was tested under uniform constant vertical load (using actual weights) and increasing monotonic lateral load up to cracking. Then, a reversed cyclic loading representing earthquake action, was used to see how much this structure can resist lateral loading after cracking.

Considerable progress has been recently done in developing nonlinear analytical models for the actual response of reinforced concrete structures. However, most of the available analytical techniques are still impractical or of limited accuracy. The work that has been done on flat slab structures has not dealt directly with the nonlinear analysis of three dimensional building frames. Recent studies have attempted to provide more accurate nonlinear analysis of structural systems subjected to vertical and lateral loads up to failure. In all these studies, the three dimensional building frame was modeled as a series of typical plane frames aligned in the direction of the applied lateral loads.

This study describes the development of computer program for the nonlinear analysis of building frames as space structures under uniform vertical and monotonic lateral load. The program is based on the finite element technique of analysis mixed with the displacement method.

The program is coded in FORTRAN language and has been tested on IBM 386 personal computer. Anisotropy, geometrical and material nonlinearities are included in the analysis. The program employs an efficient finite element that makes it possible to use a relatively small number of elements. The model is based on small deflection theory and uses a layered 48 degree-of-freedom rectangular element. The structure is treated as an assemblage of thin plate elements subjected to in plane and transverse forces.

A numerical method of analysis is developed to trace the response of reinforced concrete slabs and shells under different kinds of load conditions. The load-deflection history of such structures through the elastic, inelastic and ultimate ranges is calculated. The ultimate failure of these structures due to in-plan membrane plus bending effects is then predicted considering failures in steel and concrete along with the deterioration of the structural stiffness due to progressive cracking.

A finite element and finite strip stiffness formulations are developed to analyze reinforced concrete slab and cylindrical shell structures.

An updated Lagrangian formulation is employed to take the effects of changing structural geometry on the response of structures into account. The effect of initial stresses is incorporated by the inclusion of the geometric stiffness. Small displacements are allowed, and the theory assuming small strains and rotations is still valid at the element level.

The reinforced concrete section is modeled as a layered system of concrete and "equivalent smeared" steel layers. Full-bond is assumed to exist between the concrete and bonded steel layers. The stiffness of an element is then obtained by integrating the contribution from all the layers in the section.

Concrete behavior under the biaxial state of stress is represented by a nonlinear constitutive relationship which incorporates tensile cracking at a limiting stress, tensile stiffening between cracks and the strain-softening phenomenon beyond the maximum

compressive strength. The steel reinforcement is represented by a bilinear, strain hardening model.

As mentioned before layered formulation is adopted to represent the steel reinforcement and to simulate progressive concrete cracking through the thickness. The nonlinear variations in material properties through the depth of the slab are considered. Concrete is modeled as anisotropic nonlinear material in both compression and tension. Steel is represented as a uniaxial material with a bilinear stress-strain curve. The sources of material nonlinearity are examined and accounted for in an incremental-iterative nonlinear solution procedure. The material model for concrete allows for strain softening after cracking and crushing and gives the directions of cracks. The ability to represent cracking as a continuous process is a unique feature of the model and is significant in obtaining a good match with experimental results.

Finally, several numerical examples are presented to demonstrate the validity and applicability of the present method of analysis. The numerical results obtained from the analysis are compared to the available experimental data and the analytical results obtained by the author and other investigators and good agreement is demonstrated.

The program was used to analyze four reinforced concrete specimens tested by the author. The overall load-deflection response, cracking, deformations and failure modes are compared with the test data. A good agreement was obtained.

The test results showed a ductile behavior. Horizontal cracks at the tension side and yielding of column reinforcement was recognized. A nonlinear ductile behavior was recorded. The brittle failure by punching shear in slab was not experienced as expected due to increase the slab thickness to avoid brittle failure by punching shear of the slab and to get a ductile behavior which can compare with the analytical results from the computer program.

The test results showed that proper detailing of reinforcement over the columns can improve the deformation ability of slab-column connections, the lateral load resistance and ductility of the structure.

Analytical study of the behavioral characteristics of the reinforced concrete building frames using finite element method and framing action is presented. Correlation with the experimental results are included. The main objective of the analytical study is to provide a means of covering a wide range of parameters which can not be investigated experimentally due to budget and time constraints. To allow a meaningful analysis of analytical data and to provide a better understanding on full-scale of a reinforced concrete building frames response a parametric studies were chosen to cover a number of design variables.

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