AIN SHAMS UNIVERSITY

Faculty of Engineering

# ANALYSIS OF REINFORCED CONCRETE VOIDED SLABS

By

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# STATEMENT

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No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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To My Parents

and My Brother

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