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ANALYSIS OF REINFORCED
CONCRETE VOIDED SLABS

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By

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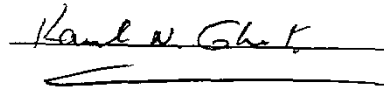


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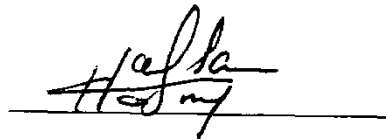
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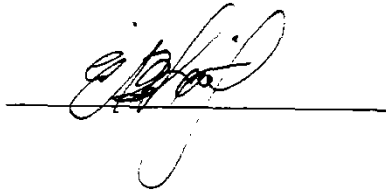
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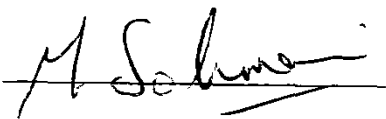
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STATEMENT

This dissertation is submitted to Ain Shams University for the degree of MASTER OF SCIENCE in Structural Engineering.

The work included in this thesis was carried out by the author in the Department of Civil Engineering (Structural Division), Ain Shams University, from November 1986 to August 1989.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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To My Parents
and My Brother

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