# TOWARDS THE REDEFINITION OF THE EGYPTIAN GEODETIC CONTROL NETWORKS:

# GEOID AND BEST-FITTING REFERENCE ELLIPSOID BY COMBINATION OF HETEROGENEOUS DATA

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#### STATEMENT

This dissertation is submitted to Ain Shams University for the degree in civil Engineering (surveying)

The work included in this dissertation has been carried out by the author in the department of public works, Ain Shams University and the department of physical geodesy, Graz university of Technology from October 1988 to January 1993.

No part of this thesis has been submitted for any degree at any other university or institution.

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#### Abstract

The geoid is very essential for accurate geodetic position computations that can meet the requirements of recent geodetic applications such as monitoring earth's crustal movements and other disciplines related to earth dynamics.

The advent of the GPS has stressed the importance of the geoidal heights. By GPS positioning, it is now possible to determine accurate ellipsoidal heights. Therefore, accurate geoidal heights are necessary to convert these ellipsoidal heights to orthometric heights which are needed in most mapping applications.

The main aim of this dissertation is the determination of a geoid and a best fitting ellipsoid for Egypt using the available heterogeneous geodetic data. This geoid and the best fitting datum are steps towards the redefinition of the Egyptian geodetic control networks.

Least squares collocation is considered the most suitable technique for geoid determination from heterogeneous geodetic data. Therefore, this technique is used for the purpose of geoid determination in Egypt.

This dissertation can be divided into four main parts. The first part, chapters 1 and 2, contains an introduction and the theoretical background required for the computations using least

squares collocation.

The second part, chapter 3, deals with the geoid determination in Egypt from the available heterogeneous data. Two solutions for the geoid determination relative to the datum of the Geodetic Reference System 1980 (GRS80) and the Egyptian datum 1907 (EGD) have been performed. The two solutions give close results especially in the areas of dense data distribution. The accuracy of the predicted undulations is about 1 - 1.5 m.

The third part, chapter 4, includes a study for the requirements for determining a more accurate geoid for Egypt in the future. The effect of smoothing the gravity field by using the geopotential models or the terrain reduction has been outlined. A study for choosing the best geopotential model for recovering the available data has been performed and new geoid solution by referring the data to this model has been computed. For the current situation of the used data, the geopotential models do not improve the accuracy.

Also the effects of the data distribution, kind and accuracy of the predicted undulation standard errors has been investigated. Decreasing the gravity data spacing improves the accuracy until a certain limit. On the other hand, decreasing the GPS data spacing improves the accuracy without any limit, providing errorless geoidal heights.

Also a simulation study for improving the accuracy of the geoid in Egypt using additional fictitious data has been

presented. Five simulation solutions based on combining assumed additional gridded gravity anomalies and /or GPS undulations with the already available geodetic data have been performed. Having a gravity net with stations about 60 km apart and GPS undulations net with stations about 120 km apart can improve the accuracy of the predicted geoidal undulation to about 0.5 m.

The last part, chapter 5, deals with the problem of determining a best fitting datum for Egypt. The required mathematical formulas for datum transformations have been derived in exact as well as approximate forms. The mean of the differences between the exact and the approximate transformed undulations is about 0.04 m which can be neglected. Therefore, the required formulae for determining the best fitting ellipsoid have been given in spherical approximation forms.

Several solutions using either the undulation condition or the deflection condition or both have been performed for determining a best fitting datum for Egypt. The undulation condition gives the best results. Also, reorienting and repositioning the ellipsoid of the GRS80 gives the best values for the summation of squared undulations and squared deflection components. Therefore, this solution has considered as the best fitting datum for Egypt. The maximum undulation relative to this datum is about 9.0 m.

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