ON THE SOLUTION OF SOME SPECIAL LINEAR PROGRAMMING PROBLEMS

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BY

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M.SC. COURSES

Studied by the Author (1975-1976)

(1)	Functional Analysis (during the whole academic year)	2 hours weekly
(2)	Theory of Groups	2 " "
(3)	Algebraic Topology	2 " "
(4)	Theory of Functions of Matrices	2 " "
(5)	The Algebraic Eigenvalue Problem	2 " "

PREFACE

The thesis consists of four chapters. Chapter I is a historical introduction to the linear programming problem and the need of it in the military, economics, and industry. This chapter exposes the linear programming problem with some examples to illustrate the linear programming model.

Chapter II deals with the solution of the linear programming problem by using the simplex method and the revised simplex method. It deals also with the simplex method with multiplicators.

Chapter III exposes the cutting problem in one-dimension; Gilmore and Gomory's method, and Bayoumi and Romanovsky's method, for solving this problem, with identical standard parent patterns, are explained.

In chapter IV we investigate the solution of the cutting problem in one-dimension with different standard parent patterns. In our investigation we make use of Bayoumi and Romanovsky's method which, in turn, depends on the simplex method with multiplicators already described in the former chapters. In this chapter, we give the texts of all our programs, and their Flow Charts.

These programs were checked on the IBM/1130 electronic computer, in the Computing Center, Ain Shams University.

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CONTENTS

CHAPTER I

THE LINEAR PROGRAMMING CONCEPT

1.1. INTRODUCTION AND HISTORICAL NOTE

Linear programming is a relatively new field in mathematics. Most of the basic work was done in 1940's by F.L. Hitchcock, L. Kantorovich, T.C, Koopmans, and G.B. Dantzig using the economic theories of J. von Neumann and W. Leontief performed in 1930's.

Linear programming is the most widely used mathematical programming technique for a number of reasons. First, the theory of linear programming is relatively easy to comprehend, and many excellent and highly readable books are available on the subject. Second, the method used to solve linear programming problems guarantees the optimal solution and is easily programmed. Third, even though many applied optimization problems are extremely complex and inherently nonlinear, the linear programming model has proved to be useful as an approximating problem in numerous instances.

In spite of its wide applicability to every day problems, linear programming was unknown before 1917. in the U.S.S.R. in 1939, Kantorovich made proposals that were neglected during the two decades that witnessed the discovery of linear programming and its firm establishment elsewhere.

The linear programming model, when translated into purely mathematical terms, required a method for finding a solution to a system of simultaneous linear equations and linear inequalities which minimizes a linear form. This problem was not known to be an important one with many practical applications until 1947.

In June 1947, intensive work began in a project titled " Scientific Computation of Optimum Programs " (SCOOP). By the end of the summer of 1947, Dantzig, who was a member of a group studying allocation problem for the U.S. Air Force formulated the general linear programming problem and developed the simplex computational method for choosing the optimal feasible program. During this period the Air Force sponsored the work performed by John Cartiss and Cohn at the U.S. Bureau of Standards on electronic computers and on mathematical techniques for solving such models. Since 1948, the Air Staff has been making more and more active use of mechanically computed programs.

Since that time the subject has received widespread attention in diverse fields as nutrition, engineering, economics, agriculture and many others. Work on linear programming problems proceeded independently until 1949, unifying the seemingly diverse subjects by providing a mathematical framework and computational method, the simplex algorithm, for formulating such problems explicitly, and determining their solutions efficiently.

Linear programming is connected with planning activities of the military. A nation's military establishment in wartime or in peace is a complex of economic and military activities requiring almost unbelievably careful coordination in the implementation of plans produced in its many departments. If one such plan calls for equipment to be designed and produced, then the rate of ordering equipment has to be coordinated with the capabilities of the economy to relinquish men, material, and productive capacity from the civilian to the military sector. These development and support activities should dovetail into the military program itself.

Besides its connection with military activities, linear programming is connected with economy and is also of wide use in industry.

In 1758 economists began to describe economic systems in mathematical terms. Mathematical economists were occupied, in the most part of their work, with the analysis of theoretical problems associated with the possibility of economic equilibria and its allocative efficiency under competitive or monopolistic conditions. During the 1930's, a group of Austru and German economists worked on generalizations of the linear technology of Walras. This work raised some questions that may have stimulated the mathematician von Neumann in 1932 in his paper "A Model of General Economic Equilibrium".

In 1947, T.C. Koopmans took the lead in bringing to the attention of economists the potentialities of the linear programming models. His rapid development of economic theory of such models was due to the insight he gained during the war with a special class of linear programming models called transportation models. At about the same time, a few other economists became interested in activity analysis and linear programming.

The Russian mathematician L. V. Kantorovich has for a number of years been interested in the application of mathematics to programming problems. He published an extensive monograph in 1939 entitled Mathematical Methods in the Organization and Planning of Production.

Turning to the use of linear programming in industry, after the second world war, there were two concurrent developments that had a profound influence ; (a) the development of large scale electronic computers, and (b) the development of inter-industry model. The latter describes inter-industry relations connected with economy. This inter-industry model was originated by Wassily Leontief in 1951.

The first and most fruitful industrial applications of linear programming is the scheduling of petroleum refinerieus which showed that there are applications by the oil industry in exploration, production and distribution as well as in refining. This is shown by Symond (1955), Manne (1956), and others.

The food processing industry is perhaps the second most active user of linear programming.

In the iron and steel industry, linear programming was used for the evaluation of various iron ores and of the pelletization of low-grade ores as shown by Fabian in 1954. In France the best program of investment in electric power was investigated by linear programming methods by Massé and Gibrat in 1957.

1.2. THE LINEAR PROGRAMMING PROBLEM

Linear programming is concerned with describing the interrelations of the components of a system. The first step constitutes regarding a system under design as composed of a number of elementary functions called "activities". The different activities in which a system can engage constitute its technology. These are the representative building blocks of different types that might be recombined in varying amounts to rear a structure that is self- supporting, satisfies certain restrictions and attains as well as possible a stated objective. Representing this structure in mathematical terms often results in a system of linear inequalities and equations; when this is so, it is called a linear programming model.

To be a linear programming model, the system must satisfy certain assumptions of proportionality, non-negativity, and additivity.

1.3. CLASSIFICATION OF PROGRAMMING PROBLEMS

There are two kinds of the programming problem, one of them is " deterministic ", which means that if certain actions are taken one can predict (a) the requirements to carry out the actions and (b) the outcome of any actions. The other is called "probablistic" in which the outcome of a given action may depend on some chance event such as the weather, government policy, or the rise and fall of customer demand.

One important way to classify programming problems is classification into multistage and non-multistage groups. Multistage models include dynamic models in which the schedule over time is a dominant feature.

A second important way to classify models is into those in which some of inputs, outputs, assignments, or production levels to be determined must occur in discrete amounts such as 0, 1, 2,... (with no intermediate amounts possible), and into those in which these quantities can take on any values over continuous ranges. The discrete problems belong to the class of nonlinear programming problems.

1.4. THE LINEAR PROGRAMMING MODEL

The linear programming problem may be stated as

Optimize $z=f(x_1,x_2,\ldots,x_n)=c_1x_1+c_2x_2+\ldots+c_nx_n$

subject to

$$g_1(x_1,x_2,\ldots,x_n)=a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n \begin{Bmatrix}\leqslant & = & \geqslant\end{Bmatrix} b_1$$

$$g_2(x_1,x_2,\ldots,x_n)=a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n \begin{Bmatrix}\leqslant & = & \geqslant\end{Bmatrix} b_2$$

$$\vdots$$

$$g_m(x_1,x_2,\ldots,x_n)=a_{m1}x_1+a_{m2}x_2+\ldots+a_{mn}x_n \begin{Bmatrix}\leqslant & = & \geqslant\end{Bmatrix} b_m$$

$$x_1,x_2,\ldots,x_n \geqslant 0.$$

The a_{ij}'s, b_i's, and c_j's are assumed to be known constants. The variables $x_1,x_2,\ldots,x_n$ are called decision variables. The function f to be optimized is called the objective function; it may be maximized with respect to the decision variables $x_1,x_2,\ldots,x_n$, such as in maximizing profit, or minimized (such as in minimizing loss). The optimization of f is carried out so that the m constraints $g_1,g_2,\ldots,g_m$ are satisfied. The constraints may be cast either as inequalities in either direction ($\leqslant$ or $\geqslant$) or as equalities (=).

Several examples follow that illustrate formulation of a linear programming model and its diverse applicability.