DECOMPOSITION METHODS IN LINEAR PROGRAMMING

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Introduction

Many decision problems are now formulated as linear programs, requiring the maximization or minimization of a linear objective function subject to linear constraints. Such programs often have special structure. The nonzero elements of the constraint matrix may appear in diagonal blocks, except for relatively few rows. Such structure is almost always found in truly large problems, since these commonly arise from linking of independent subunits. Decomposition methods are developed to take advantage of this structure to achieve significant gains in computational efficiency and reduction in computer memory requirements. Such methods are justified only for truly large problems, which cannot otherwise be solved because of time and/or storage limitations.

National Planning is a field which suggests the application of decomposition methods. The national economy is usually divided into several sectors. Within each sector there are many constraints which are unaffected by the activities of other sectors, but there are few constraints and common objectives that tie all the sectors together.

In this thesis we try to present the formulation of the problem of the socio-economic development, on the national level, into a large-scale and multicriteria linear programming problem. We also present a method for its solution using the direct decomposition method. This is done in chapter 3. We consider several objective functions simultanionsly to reflect the different aspects of socio-economic development. In general the requirement that all functions be maximized at the same time is a contradictory one. Generally the maximization of one of the objective functions will cause a deviation of at least one other function from its maximum. We thus have to find the feasible solution that can be considered the best regarding the maximization of all objective functions. We shall consider the optimal solution to be that solution for which the summation of the relative deviations of the different objective functions from their optima is minimum. This turns to be the optimal solution of an overall objective function equal to the summation of the different objective function each one weighted by the reciprocal of its optimal value.

In Chapter 4 we present a program package for the method. The program is written in FORTRAN language for the general case with N0 objective functions and K sectors. The package is self documented. In writing the program we prefer to save storage on the expense of more calculations. But always these calculations are simple and few. Flow charts are also presented. The program can solve problems that have a starting basic feasible solution, i.e., those for which the elements of the constant column are nonnegative.

In chapter 2 we give a review to 4 different decomposition methods. The first is Dantzig-wolfe method which is the first one to be developed and the most well known and used. The method consists of a finite number of steps. It can be considered the most efficient one since a phase I procedure can be applied to it in order to obtain a starting basic feasible solution when the evident starting solution is not feasible, similar to the simplex method of linear programming. It was the publication of the Dantzig-Wolfe decomposition principle in 1960 that initiated the studies in this field. We next review Grabowski vertical decomposition method which solves the problem through three steps. In the first step it solves a parametric linear programming problem for each sector by using Jordan-Gauss and Fourier-Motzkin elimination methods for solving a system of linear constraints. In the second step a linear programming central problem is formed and solved by the simplex method. The third step is simple substitution of the results of the second step in the solution of the first step to obtain the solution of the original problem. The method is quite interesting from the theoretical point of view. But in application the problem of step 2 may be very large, it even could be larger than the original problem. The third reviewed method is the direct decomposition method which we choose to apply to our multiobjective problem. We choose it because it is the simplest one, and it works with the whole matrix of coefficients for one time while Dantzig-Wolfe method solves sector problems several times and coordinates different solutions through a master problem that is to be solved several times also. For a problem of K sectors, the direct decomposition method saves about $\frac{K-1}{K}$ of the storage capacity that is needed in case of solution with the simplex method. It has the disadvantage that it solves only a problem c.. which the initial basic solution is feasible. Lastly we give a short review to Kornai-Liptak method. The method was developed in 1962 in Hungary to solve the two level planning model of the 1966/70 plan. The model was built to determine the targets of production and foreign trade for 1970, the terminal year of the plan period, and the pattern of investments activities for the period 1966 to 1970. This method however, has proved to be interesting only from the theoretical point of view; practically, its convergence is very slow and Dantzig-Wolfe method has been used instead of it for practical application in Hungary.

Chapter 1 is an introductory one, it demonestrates the importance of decomposition methods in national planning.

The material in Chapter 3 as it stands has been accepted for publication in the Hungarian Journal SZIGMA.

CHAPTER (1)

The Importance of Decomposition Methods in National Planning

1.1. Introduction:

Since the second world war there is a strong direction towards consumption by the people everywhere. As society developed and its standard increased, the ratio of industralised material increased, because the human need for basic necessities of life such as food and cloth reaches saturation at a certain level, but other services and consumption goods, which mostly need industrial knowledge and production, have an open and extended range before them. Fulfilment of the consumption aims of the people becomes one of the basic obligations of the modern state.

A great problem faced by the developing countries is that its ability to increase production especially industrial production is far behind the consumption attitude of the people. A limit should then be put to the consumption attitude to make it approximately equal to the ability to increase production, either in agricultural or industrial products. This is the basic reason which made thinking, in post war years, to look at the society as a whole in matters of production and consumption and this creates the idea of national planning. In national planning, we get a comprehensive view that covers a period of five years, or less or more, to draw a picuture indicating the directions of production and consumption and showing the different kinds of economic, social and political relations.

National planning is a technique that can be defined as an application of the scientific method in thinking to the problem of socio-economic development on the national comprehensive level, including not only material elements but also social and political elements as well.

Decision making is the practical expression of power, which is according to any political system concentrated in the hands of a few, who should take the decisions necessary to lead the society forward. National planning tries on a scientific basis to help in making correct decision in accordance with the objectives and ideals of society. National Planning, like any scientific technique, depends first on observations, then on hypothetical solutions in the form of model or theory. Tests prove the validity of the hypothesis, then theories are changed accordingly. Experiments and observations are repeated day after day untill we find an acceptable theory.

National planning implies full utilisation of all productive resources of the country. It also implies certain basic objectives of which the national plans are instruments of realisation. In general terms, the basic objective of planning may be stated as the most rapid increase in the material and cultural standard of living of the people, possible under existing historical circumstances. The objectives of planning may be stated as:

1. rapid industralisation of the country.
2. modernisation of agriculture.

These two objectives are interrelated among each other, one cannot be realised without the other. Industralisation requires a substantial increase in marketable agricultural output in order to feed the increasing non-agricultural population. This requires modernisation of the methods of agricultural production, agricultural machinary and implements, tractors, and fertilizers, etc. These must be produced by industry.

In socialist and developing economies, planning is more essential than in capitalist economies. This is because in these socialist economies there does not excist a labour market , in the precise sense, nor a capital market. Thus the market mechanism is replaced by planning in running and developing these economies. It determines the conditions of reproduction and distribution of social products.

1.2. The Use of Mathematical Models in Planning:

Just as those engaged in other social sciences, the economist too, is inclined to envy the scientist for his being able to resort to experimentation. The scientist is in a position to check his hypothesis by means of experiments which can be repeated as many times as necessary. Now, mathematics has provided a tool which brings our discipline also in this respect nearer to the natural sciences. The natural scientist would change the conditions of his experment in accordance with certain purposeful principle and would register the reaction of the test subject to the changes. We are proceeding in a similar manner when working our program in several variants. Changing the conditions of the experiment (the model's framework and numerical values) in accordance with certain purposeful principles, we would register the reaction of the test subject: the optimal program. The character of the experiment will, of course, differ in many respects from that of emperical experimentation in the field of natural sciences. Our experiments will be carried out on paper only, by means of figures. The two types of experiment series have, non the less, some important analogous characteristics such as the recording of the effects of systematic changes in the conditions, the numerical measuring of these effects and the possibility of repetition.

The mathematical models are not claimed to be substitutes for the traditional (non mathematical) methods of planning. A kind of cooperation must be established between the traditional and mathematical planning methods. The mathematical models used in planning can reveal possible contradiction and equilibrium failures between the targets of the plan worked out by the traditional methods. The planners are not always able with the traditional planning methods to survey and coordinate all the mutually dependent trends as consistently as is possible with the constraints system of the mathematical programming model.

The first mathematical technique used in planning may be input-output analysis. It is one of the most important tools of planning. It covers the purpose of establishing the quantitative relations between various branches of production which must be maintained in order to assure a smooth flow of production in the national economy. It studies the conditions of mutual consistency of the inputs of various branches of the national economy which result from the fact that the output of one branch is the source of input in other branches. In all countries which have recourse to long term planning, production targets are set with the aid of a system of relations which is identical or similar to an open Leontief input-output model. This is explicitly recognized in western countries. In soviet-type economies, the accent is laid on "material balance-sheets" for input and output quantities. But this method is basically little different - if at all - from that originated by Leontief.

Another important tool is linear programming which has the advantage of introducing the optimization principle to the plan objectives. The problem of national planning can be formulated as a linear programming problem. We may either prepare a highly aggregated programming model, in which case the freedom of choice is extremely narrow and the extreme aggregation and excessive simplification endanger the utility of the computed results, or else we work out a very large scale model that is free from these deficiencies, but in this case the numerical computing of the problem cannot be carried out, even with the most powerful electronic computers. Solution of such large scale problems can be made by decomposing them into several linear programming problems of moderate size and using decomposition methods to solve them.

Dynamic models are recently employed in socialist economics. A dynamic model offers a number of advantages as compared with a static model which does not ensure the organic coordination of the gross output plan with the plan of investments. In a static model of the inter-sector balance productive investments are regarded as given, with their subsequent specifications in an iterative way. In a dynamic model productive investments, linked with the expansion of output, are ascertained as a result of the solution of