OPTIMAL CONTROL AND ESTIMATION FOR STOCHASTIC DISTRIBUTED-PARAMETER SYSTEMS

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Abstract

This thesis is concerned with computational methods for solving both optimum distributed control and state estimation problems for Stochastic Distributed-Parameter Systems. The systems considered are represented by linear partial differential equations with a distributed stochastic disturbance.

For the control problem a direct method is used, in which the control functions are represented by a double convergent series of orthonormal functions in both space and time, to derive a scheme for obtaining the optimum feedback distributed control when knowing the state of the system. No magnitude constraints are considered on the control and a quadratic performance index is considered.

For the problem of state estimation a mean square error criterion is considered and the number of measurement transducers is finite. The elements of the system Green's function, gain function, and error covariance matrices are expanded by means of a set of orthonormal functions. Recursive formulas, are derived, to obtain the optimum gain and error covariance matrices.

Two examples are solved: one for the optimum distributed control of a one-dimensional system described by the heat diffusion equation, and the second for the state estimation of the neutron flux distribution in a slab-type nuclear reactor with spatial disturbance.

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Chapter 1
INTRODUCTION

The optimal design and control of systems and industrial processes has been of concern to the applied scientest and engineer during recent years. High accuracy of operation of a system or a unit and also high speed of operation are often required with the least expenditure of available means.

It is necessary, to develop and create methods of control and design that will enable us to utilize to the fullest all the potentialities of the systems and to create systems that are optimal in some definite sence.

Many of the problems of control and design in airframe, shipbuildings, electronics, nucleonics, and other engineering fields are, in essence, problems of control for systems with distributed parameters. The dynamic behaviour of such systems is described by partial differential equations, integral equations, integro-differential equations, and occasionally by more general and more complex functional relations.

In engineering and scientific applications it is often necessary to device optimal control for a complex system whose state is charecterized by one or several parameters distributed in space. To this class of systems belong many industrial processes such as :

1. Heating of metal for rolling or heat treatment in continuous or rapid-heating furnaces.
2. Drying and firing of friable materials in rotary furnaces.
3. The diffusion of neutrons in atomic piles.
4. Diffusion of concentrations of liquid or gaseous substances in chemical processes.

In such processes the material moves in space through processing zones where various types of fields acting on the material,(thermal, electrical, chemical, etc.), are distributed in both time and space.

However, under certain conditions, high-order, lumped-parameter systems can be approximated by a system with ditributed parameters. For example, the equation for a system consisting of a large number of cascaded blockes whose impulsive response is nonoscillatory can be approximated by the heat conduction equation.

In spite of the well-developed theory of optimal systems with lumped parameters and the availability of powerful means of realizing such systems, there are few examples of their practical implementation. this is partially because real physical objects requiring optimal control devices are complex units that simply cannot be described in terms of ordinary differential equations.

Therefore, it becomes necessary to develop the theory of optimal systems further and to generalize it to systems with distributed parameters. This direction is of great importance in many engineering applications.

Now we will discuss breifly three examples to show the necessity of treating some physical systems as distributed parameter systems, and how to formulate the control problem.

§ Nuclear Reactor Systems [1]

Recent technical advances in research reactors and the successful long-time operation of the relatively small experimental power reactors, have successfully laid the foundation for the future of nuclear power generation from large power reactors, and consequently the study of the dynamical behavior of large reactors becomes essential (e.g., stability analysis and optimal control of reactors).

Most reactors built in the past often used the classical point-model-reactor-kinetics equation, which approximates the spatially dependent reactor system as a space independent system by employing certain spatial averages of the system variables. When the reactor in question is small, the point model is usually adequate for the control and stability analysis since the assumption that the system responses are space-time sperable is valid.

However, for the much larger reactors, the neutron flux in the various regions of an inhomogeneous reactor core can become loosely coupled and any nonuniform changes in the properties of the reactor cause local distortions of the power distribution. In this case, the space-time sperable assumption is no longer valid, and the use of the point model in the study of spatially-dependent reactor systems is open to question, as the error involved by using the point-mod l approximation increases with the size of the core.

Similary, if an optimal control system was designed to have uniform burnup using the point-model, the system may indeed be optimal w.r.t. its model, but it may not be optimal in the true sense because no matter how good the design is, it is only as good as the mathematical model.

The above indicates that, it is not only desirable but necessary to study the control and stability of the large reactor systems by taking into account the spatial distribution of the system responses.

A linear reactor model, for example, is assumed to be an infinite slab reactor (one spatial dimension) with a width h. Two infinitesimally thin control rods are located at x_1 and x_2, respectively. The configuration of the system is shown in Fig. 1-1

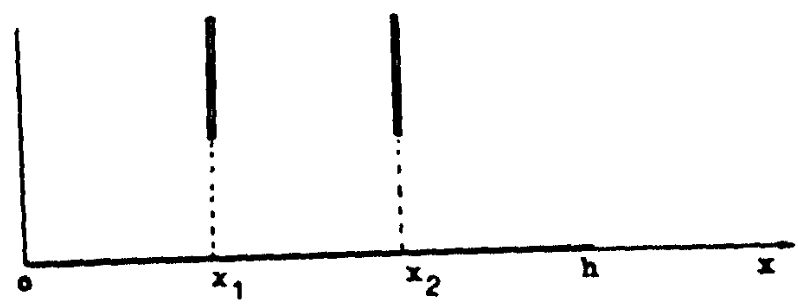

Fig. 1-1 Infinite Slab Reactor and Control Rods Configuration.

The system is described by,

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} + cu + f_1(t)\ \delta(x - x_1) + f_2(t)\ \delta(x - x_2) \qquad (1\text{-}1)$$

or generaly it can be put in the hypothetical form :

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} + cu + M(x)\ F(t,x) \qquad (1\text{-}2)$$

where a is diffusion coefficient.

c is neutron fission rate.

f(t) is effective absorption cross section.

The allawable boundary conditions for the system are :

1. The neutron flux vanishes at the extrapolated boundaries,

$$u(t,0) = u(t,h) = 0 \qquad (1-3)$$

2. A certain linear combination of the current (gradient of the flux)and the flux vanishes at the boundary.

We have two main problems, the optimum regulator problem where the performance index has the form,

$$J = \int_0^{t_1}\int_0^{h} |u - u_d| \; dx \; dt + \int_0^{t_1}\int_0^{h} F^2 \; dx \; dt \qquad (1-4)$$

where u_d is the desired state of the system.

The second problem is the time optimal control, where the performance index has the form,

$$J = \int_0^{t_1} dt \qquad (1-5)$$

Sometimes a control constraint is imposed so that,

$$\| F(t,x) \| < k \qquad (1-6)$$

§ Continuous Heating Furnaces [2]

In these furnaces the material is heated while being moved through the processing zones.

Fig. 1-2 shows a schematic digram of a three-zones continuous kiln. In the continuous and welding zones two-sided heating of the slabs takes place and in the soaking zone one-sided heating takes place. The kiln serves for heating slabs before rolling in a hot strip mill. The temprerature in the working space of the kiln is measured with the aid of temperature sensors.

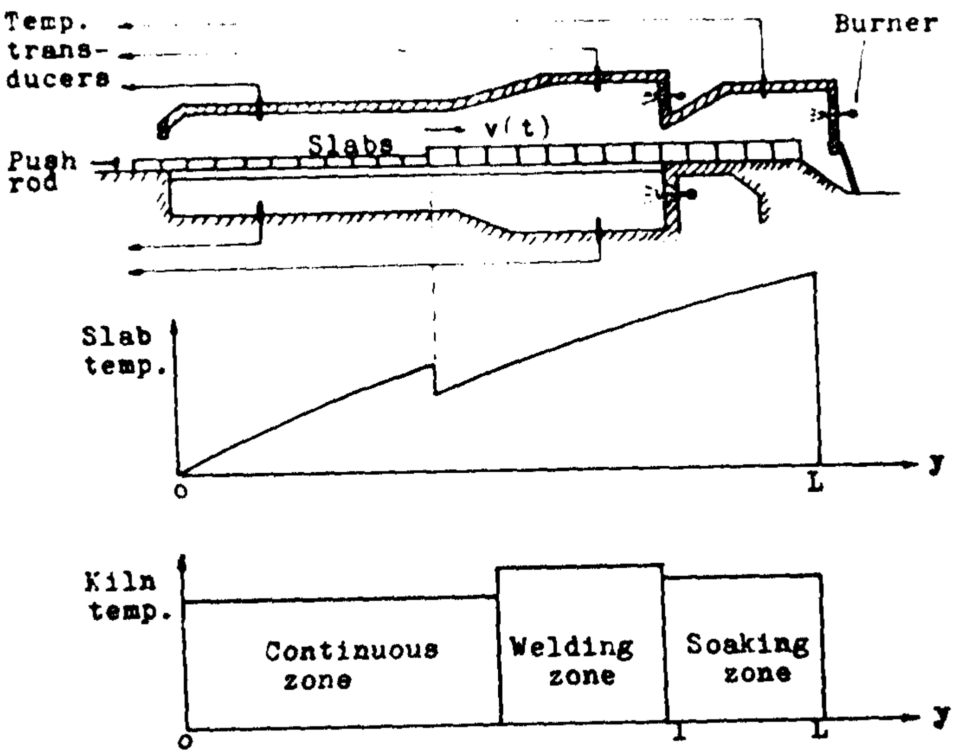

Fig. 1-2 Three-zone Continuous Kiln with Bottom Heating.

The temperature of the slabs at the furnace outlet depends on the character of the change in the temperature ditribution in the furnace zones during the time a given slab of the material remains in the furnace, as well as on the character of the change of the velocity of the material v(t) during the same time, the thickness of the slab, its thermal conductivity, heat capacity, density, etc. In other words, the temperature of the slabs at the outlet depends on the entire history of heating from the moment of entry to the moment of exit from the furnace.

The kiln is provided with temperature regulators for the soaking, upper and lower welding zones. Variation of assignments using these regulators represents the control action and is realized by the welding operator in order to obtain the specified heating of the slabs at the output from the kiln.

The optimal control system can be devided functionally into two parts, model of the object which calculates the distribution of temperature of the metal along the length of the kiln, and the controlling device properly acting on the setting of the temperature regulators in the different zones.

The model of the object obtains information from the sensors mounted on the kiln, as shown in Fig. 1-3

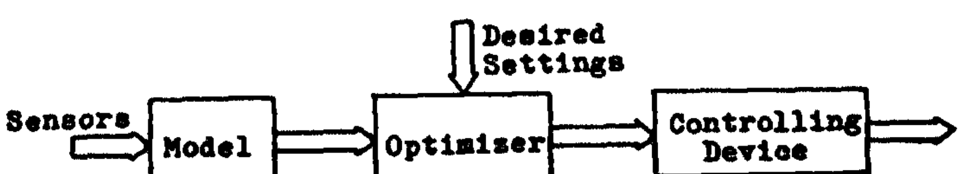

Fig. 1-3 Optimal Controller for a Continuous Kiln.

In the process of heating the slab in the kiln we will neglect the heat transfer along the direction of motion of slabs due to loose connections of adjacent slabs and the small temperature gradient along the direction of their motion. We shall also neglect the heat exchange which takes place from the end faces of the slabs because their area is small in comparison with the area of the remaining heating surface of a slab. Under these conditions we obtain the problem of heat conductivity for an infinitely wide plate of thickness S moving with velocity v which depends on time t in the positive direction of the y axis.

Let x and y be the spatial coordinats which are rigidly associated with kiln and $0 \leqslant x \leqslant S$, $0 \leqslant y \leqslant L$, where S is plate thickness and L is kiln length.

Using these coordinates, the heat equations have the form,

$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} - v \frac{\partial u}{\partial y} \qquad (1\text{-}7)$$

where the function $u = u(t,x,y)$ characterizes temperature distribution in the metal $0 \leqslant x \leqslant S$, $0 \leqslant y \leqslant L$ during time t, $0 \leqslant t \leqslant T_1$. (Here a is the temperature conductivity coefficient)

The main equation is supplemented by boundary conditions which characterize the heat exchange between the metal surface and the working space of the kiln by radiation and convection:

$$-\lambda \frac{\partial u}{\partial x}\Big|_{x=0} = \sigma_1' \left\{ \left[q_2(t,y)\right]^4 - \left[u(t,0,y)\right]^4 \right\} + \alpha_1' \left[q_2(t,y) - u(t,0,y)\right], \quad 0 \leqslant y \leqslant l, \; 0 \leqslant t \leqslant T_1 \qquad (1\text{-}8)$$

$$-\lambda \frac{\partial u}{\partial x}\Big|_{x=S} = \sigma_1'' \left\{ \left[q_1(t,y)\right]^4 - \left[u(t,S,y)\right]^4 \right\} + \alpha_1'' \left[q_1(t,y) - u(t,S,y)\right], \quad 0 \leqslant y \leqslant L, \; 0 \leqslant t \leqslant T_1 \qquad (1\text{-}9)$$

where l is total length of the continuous and welding zones, σ'_1, σ''_1 and α'_1, α''_1 are respectively the radiation and convection coefficients, q_1 and q_2 are temperatures of heating media, λ is the coefficient of thermal conductivity.

For the soaking zone where the metal is heated one-sidedly only from above the condition,

$$\frac{\partial u}{\partial x}\bigg|_{x=0} = 0 \qquad , l < y < L, \; 0 < t < T_1 \qquad (1-10)$$

is satisfied and this corresponds to the case of no heat flow through the lower surface of the slab.

The boundary condition which takes place at the input into the kiln for y=0 assumes the form,

$$u(t,x,0) = \rho_0 = \text{constant} \qquad , 0 < x < S, \; 0 < t < T_1 \quad (1-11)$$

where ρ_0 is the ambient temperature.

The initial condition is specified in the form,

$$u(0,x,y) = u\,(x,y) \qquad , 0 < x < S, \; 0 < y < L \qquad (1-12)$$

where u (x,y) is a known function.

After the slab emerges from the kiln ready for rolling we can assume that from this moment, there takes place cooling of the infinitely wide plate of certain thickness S'. The equation for internal heat change has the form,

$$\frac{\partial u_1}{\partial t} = a\,\frac{\partial^2 u_1}{\partial x^2} \qquad , 0 < x < S', \; 0 < t < T_2 \qquad (1-13)$$

The boundary conditions can be written in the form,

$$-\lambda \frac{\partial u_1}{\delta x}\Big|_{x=0} = \sigma_2' \left\{ \rho_0^4 - [u_1(t,0)]^4 \right\} + a_2' [\rho_0 - u_1(t,0)] \quad (1-14)$$

$$-\lambda \frac{\partial u_1}{\delta x}\Big|_{x=S'} = \sigma_2'' \left\{ \rho_0^4 - [u_1(t,S')]^4 \right\} + a_2'' [\rho_0 - u_1(t,S')] \quad (1-15)$$

with initial condition,

$$u_1(0,x) = u_1(T_1,x,L) \quad (1-16)$$

where T_1 is the moment when the cooled slab emerges from the kiln.

The problem of optimal heating is to find a law of change with time in the temprature of the heating medium (furnace) to ensure in a fixed time a minimum deviation, in any definite sense, of the temprature distribution in the billet from a given distribution.

§ Chemical Reaction with Radial Diffusion [3]

In any realistic consideration of a pocked tabular reactor it will generally be necessary to take radial variations of temprature and concentration into account, requiring even for the steady state a describtion in terms of independent variables. The study of optimal heat-removal rates in such a reactor illustrates as well how an optimization study can be used in arriving at a practical engineering design.

In describing the two-dimensional reactor it is convenient to define x(t,z) as the degree of conversion and y(t,z) as the true temprature devided by the feed temprature.