

Ain Shams University
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**ULTIMATE BUCKLING LOAD OF STIFFENED
NON-RECTANGULAR
PLATES WITH INITIAL IMPERFECTION**

by

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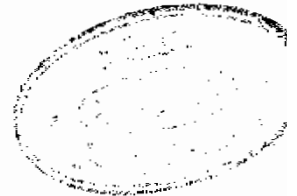
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TO MY MOTHER

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REFERENCES

STATEMENT

This dissertation is submitted to Ain Shams University for the degree of the MASTER OF SCIENCE in Structural Engineering.

The work included in this thesis was carried out by the author in the department of Structural Engineering, Ain Shams University, from October 1990 to September 1993.

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LIST OF SYMBOLS

a, b_1, b_2	parameters defining the plate dimension .
$b_{av.}$	average width of the plate .
c	opening size .
D	flexural rigidity of the plate .
E	modulus of elasticity .
e	opening eccentricity .
G_r	shear modulus of elasticity .
h	plate thickness .
K	dimensionless buckling coefficient .
k	shear correction factor .
N_k	shape function corresponding to nodal point k .
N_x, N_y	normal force in the global X,Y directions. (stress resultants) .
M_x, M_y, M_{xy}	moments in the global X,Y directions . (stress resultants) .
P	concentrated load acting on the plate .
p, p^*	distributed loads acting on the plate .
P_{cr}, P_{cr}	critical buckling load (distributed and concentrated) .
Q_x, Q_y	shear force in the global X,Y direction . (stress resultants) .
U, V, W	global displacements in the X,Y,Z directions .
$v_{1k}^x, v_{1k}^y, v_{1k}^z$	components of the nodal coordinate v_1 at node k in the global X,Y,Z directions respectively .

LIST OF SYMBOLS 'd

$v_{2k}^x, v_{2k}^y, v_{2k}^z$	components of the nodal coordinate v_2 at node k in the global X,Y,Z directions respectively .
$v_{3k}^x, v_{3k}^y, v_{3k}^z$	components of the nodal coordinate v_3 at node k in the global X,Y,Z directions respectively .
X_1, X_2, X_3	global coordinate of any point .
$\bar{X}_1, \bar{X}_2, \bar{X}_3$	unit vectors in the global X,Y,Z directions .
x_1^*, x_2^*, x_3^*	local coordinate of any sampling point within the element .
$\bar{x}_1^*, \bar{x}_2^*, \bar{x}_3^*$	unit vector in the local coordinate directions.
α	ratio between buckling coefficient of trapezoidal plates to the coefficient of rectangular plates .
β_{1i}, β_{2i}	rotation of the normal at any point (i) .
$\epsilon_{x^*}, \epsilon_{y^*}, \epsilon_{z^*}$	normal strain components defined with respect to local system of axes .
$\nu_{x^*}, \nu_{y^*}, \nu_{z^*}$	shear strain components defined with respect to local system of axes .
$\sigma_{x^*}, \sigma_{y^*}, \sigma_{z^*}$	normal stress components defined with respect to local system of axes .
$\tau_{x^*y^*}, \tau_{x^*z^*}, \tau_{y^*z^*}$	shear stress components defined with respect to local system of axes .
ν	Poisson 's ratio .
ξ, η, ζ	curvilinear coordinate of any point within the element

LIST OF SYMBOLS 'd

ξ_k, η_k, ζ_k	curvilinear coordinate of nodal point k for element.
γ, β	ratios defining the plate geometry.
δ	opening size to plate average width ratio.
ω_i	weighting factor for sampling point (i) .

LIST OF MATRICES

$\langle a_i^n \rangle$	displacements and rotations vector corresponding to load increment n at the i th iteration .
$\langle \Delta a_i^n \rangle$	the required change in displacements and rotations vector to minimize the residual force .
$[B]$	strain - displacement matrix .
$[B_i]$	contribution of node i to the strain displacement matrix .
$[B_L]$	linear part of the strain displacement matrix .
$[B_{NL}]$	non-linear part of the strain displacement matrix .
$[D]$	elasticity matrix .
$\langle F^e \rangle$	nodal force vector .
$\langle F_P^e \rangle$	nodal force vector equivalent to the body force P per unit volume .
$\langle F_t^e \rangle$	nodal force vector equivalent to the surface force t per unit area .
$\langle F_{\epsilon_0}^e \rangle$	nodal force vector equivalent to initial strain .
$\langle f^n \rangle$	external applied force vector at load increment n .
$[G]$	matrix related to the geometric stiffness and strain displacement matrices .
$[J]$	Jacobian matrix .
$[K]$	tangential stiffness matrix .
$[K_0]$	linear part of the stiffness matrix .
$[K_\sigma]$	geometric stiffness matrix .

LIST OF MATRICES 'd

$[N]$	matrix relating displacements and rotations at any point to the nodal displacements and rotations.
$[N_k]$	contribution from node k to the global matrix N.
$\langle P \rangle$	body force per unit volume vector .
$\langle p_i^n \rangle$	internal force vector corresponding to load increment n at the ith iteration .
$[S] , [R]$	matrices containing the partial derivatives of w with respect to x and y .
$\langle t \rangle$	in-plane force per unit area vector .
$\langle X \rangle$	global coordinates vector .
$\langle \bar{X} \rangle$	unit vectors in the global X , Y , Z directions .
$\langle x_1 \rangle , \langle x_2 \rangle , \langle x_3 \rangle$	local coordinate vectors .
$\langle \bar{x}_1 \rangle , \langle \bar{x}_2 \rangle , \langle \bar{x}_3 \rangle$	unit vectors in the local x , y , z directions.
$\langle u_1 \rangle$	displacements vector at any point within the element .
$\langle u_{ik} \rangle$	displacements at nodal point k .
$\langle u_* \rangle$	virtual displacements resulting from virtual nodal displacements .
$\langle v_{1k} \rangle , \langle v_{2k} \rangle , \langle v_{3k} \rangle$	nodal coordinate vectors .
$\langle \bar{v}_{1k} \rangle , \langle \bar{v}_{2k} \rangle , \langle \bar{v}_{3k} \rangle$	unit vectors in the nodal coordinate directions .
$[\theta]$	transformation matrix between local and global coordinate sets .
$\langle \psi_i^n \rangle$	residual force vector corresponding to load increment n at the ith iteration .
$\langle \epsilon \rangle$	strain components vector .