THE ELECTRICAL CONDUCTIVITY AND ITS EFFECT ON THE STABILITY OF A CYLINDRICAL JET

A THESIS

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AHMED ELAZAB ALI RADWAN

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SUMMARY

The thesis is mainly concerned with the linear and non-linear hydrodynamic, magnetohydrodynamic (MHD) and gravitodynamic (GD) instabilities of different cylindrical fluid surfaces.

The problems under consideration have some practical applications in the industrial and astrophysical domains.

In chapter I, we discuss the concept of MHD and GD stabilities. We introduce some of previous hydrodynamics, MHD and GD studies of jets instability. The nonlinear perturbation techniques those used in solving stability problems, in general, are briefly discussed. The linear and nonlinear techniques which are used in solving the problems of the present work are sketched.

Chapter II is devoted to MHD stability of a perfectly conducting jet under the inertia and electromagnetic forces. The fluid is penetrated by homogeneous magnetic field and ambient with force-free magnetic field. The equilibrium state is studied, the Bessel function model is obtained and the fluid equilibrium pressure distribution is identified. The perturbation state has been investigated and the desired stability criterion is derived. The latter is studied for some simplified cases as well as for the general case. The magnetic field inside the fluid has always a stabilizing

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influence. The force-free magnetic field has stabilizing effects due to the seperate terms of its equilibrium components. On the other hand, it is stabilizing in some regions and destabilizing in others due to the term resulting from the interaction of the Bessel function model components. The required restrictions for the stability are determined. These restrictions are extensively changed if the fluid is not perfectly conducting. The asymptotic expression of the dispersion relation, for very long wavelengths and at the same time the current density-field ratio bieng very small, is obtained. Several reported results are recovered as limiting cases from the present results.

Chapter III deals with the stability of a cylindrical interface between cylindrical gas jet and a non-infinite (radially) liquid influenced by the capillary, gas inertia and liquid inertia forces. The fluids are assumed to be inviscid, incompressible and of uniform densities. The equilibrium state as well as the perturbed one is studied. The problem is converted to a boundary-value problem in two adjoining regions with a common boundary. A non-singular solution for each region is obtained. The solutions are matched at the interface and a general dispersion relation is derived and studied to all (axisymmetric and non-axisymmetric) modes of perturbation. The stability and instability states are identified. Numereous stability criteria are obtained as limiting cases from the present

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criterion. A comparison is made between the present results and those of the full fluid jet ambient with vacuum. A physical interpretation is given for the similarities between the results of these models. The model under consideration is stable to all non-axisymmetric modes for all wavelengths and all densities ratio values. To axisymmetric mode, for all densities ratio values, it is stable if the perturbed wavelength λ is equal to or longer than the circumference $2\pi R_0$ of the gas jet, while it is only unstable as long as $\lambda < 2\pi R_0$.

In chapter IV, we have presented the MHD stability of a perfectly conducting compound jet (i.e. liquid jet of density ρ immersed in a different liquid of density ρ') subjected to the combined effects of the inertia, capillary and electromagnetic forces. The dispersion equation (valid to all modes $m \geq 0$) is established. Enormous recent results are recovered as limiting cases. Analytical and numerical stability studies has shown that the capillary force has exactly the same character on the instability of the model as in the case with $\rho' = 0$. The magnetic field has always a stabilizing effect to all modes ($m \geq 0$) for all wavelengths and all (ρ'/ρ) values. Above a certain value of the field intensity H_0, the capillary instability is completely suppressed, for all wavelengths and all (ρ'/ρ) values, and then stability sets in. This may be due to the fact that the fluids are perfectly conducting. The electrical

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conductivity of the fluids has advantage that the capillary instability can be suppressed if the magnetic field intensity is so strong enough.

The nonlinear theory is developed and the second order terms for each fluid are obtained. Our recent nonlinear results as $\rho=0$ or $H_0 = 0$ (1986) are recovered and also the very simple case of Callebaut (1971), if both $\rho'=0$ and $H_0 = 0$, is recovered. The second order terms are studied analytically and numerically in details. To axisymmetric mode m=0, for very long wavelengths, some coefficients of the second order terms are singular. These singularities are expected to be disappeared if the fluids are assumed to be non-conducting or/and viscous. The nonlinear terms give the corrections to the linear theory and determine the domain of validity of the linear theory.

Finally in chapter V, the stability of a self-gravitating fluid jet (of density ρ) embedded in a self-gravitating medium of different density ρ' has been investigated. The relevant perturbation equations are solved and the required dispersion relation is derived. The analytical results of the stability discussions are confirmed numerically for different values of (ρ'/ρ). The largest domain of instability occurs when m=0 as $\rho'=0$, then that domain shrinks fastly with increasing the $0<(\rho'/\rho)<1$ value. If $\rho=\rho'$, there is no dispersion !. As $\rho'>\rho$ the model is unstable for

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all wavelengths. If $\rho' = 0$ we recover Chandrasekhar's results (1981) for a self-gravitating full fluid jet in vacuum.

The relevant nonlinear perturbation equations are solved for each fluid. The second order terms are calculated for non-axisymmetric modes $m \geq 1$ as well as axisymmetric one $m=0$ for all values of (ρ'/ρ). The second order coefficients are studied, the limiting cases are obtained and the behaviour of the second order terms is analyzed.

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CHAPTER I

INTRODUCTION

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CHAPTER I

INTRODUCTION

1- THE AIM OF THE PRESENT WORK AND ITS APPLICATIONS:

The main preprogative of the present work is to investigate a couple of magnetohydrodynamics stability problems of different cylindrical surfaces and to identify the effect of the electrical conductivity on the instability character.

The contents of this thesis is mainly classified into two parts. In the first part the magnetodynamic and hydrodynamic stability of different cylindrical surfaces is developed. The first class of this part is concerned with the magnetodynamic stability of a perfectly conducting jet influenced by the inertia and electromagnetic forces. The jet is acted upon force-free magnetic field i.e. Bessel function model.

In the second class of the first part we have presented the hydrodynamic stability of cylindrical interface of fluid jet submerged in a liquid of different density. The liquid surrounding the fluid jet is terminated by solid cylindrical surface of radial distance qR_0 $(1 < q < \infty)$ where R_0 is the radius of the jet. The system is endowed with surface tension and influenced by the fluids inertia forces.

The second part of the thesis is devoted to the linear and nonlinear magnetohydrodynamic (MHD) and gravitodynamic

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stability of a compound jet acted upon inertia, capillary, electromagnetic and self-gravitating forces.

It has been known since a turn of century that circular static non-viscous liquid jets are capillary unstable to axisymmetric mode of disturbances having small axial wavenumbers. Observation has shown that this capillary instability, driven by surface forces on the jet interface, leads to the breakup of the jet into droplets.

This knowledge has been essential for many applications ranging from the design of sprays to the design of inkjet printers.

In the present era in particular in the last few decades the hydrodynamic, electrohydrodynamic, magnetohydrodynamic and gravitodynamic stability of different cylindrical configurations received the attention of many investigators. This is due to its practical applications in numereous fields of science, e.g. industrial field, geophysics, astrophysics, ... etc.

2- THE CONCEPT OF STABILITY :

In order to understand the stability and instability concepts, we consider a dynamical system in a stationary state i.e. no any variable is function of time in this state. If X_1, X_2, ..., X_n are those parameters which define that system, then X_j ($j= 1,2,....,n$) may be the wavenumbers,

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