

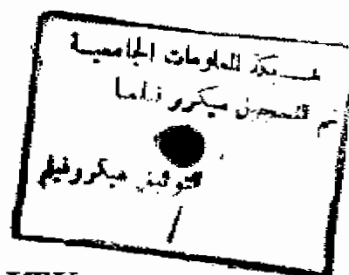
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LIMIT THEOREMS FOR SOME MEMBERS OF THE ORDER STATISTICS

THESIS

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Introduction

INTRODUCTION

Order statistics is found to play a fundamental role in non-parametric problems from the theoretical and practical points of view. Theoretically, a considerable amount of new statistical inference theory is established from order statistics. Practically, statistical inference based on order statistics provides us with procedures that are simple and broadly applicable.

The main objective of the present thesis is to study the limit theory of order statistics specially the limit theory of central order statistics. This theory is mainly concerned with the limiting distributions of the central members of order statistics.

Many authors tackled the limiting distributions of the members of order statistics, notably, Tippett and Fisher [20], Gnedenko [7], Smirnov [14] and Galambos [4]. The complete solution to the problem of the limiting distributions of the maximal term and minimal term was given by Gnedenko [7] who determined the class of the limiting distributions $\frac{\xi_n^{(n)} - b_n}{a_n}$, $a_n > 0$, b_n are suitable constants of the maximal members. Tchebycheff [17] studies the class of the limiting distributions for

the intermediate members of the order statistics under some specified conditions, these conditions are dropped by Smirnov [15] who determined the limiting distributions of the intermediate members of order statistics. Smirnov[14] completely found the class of the limiting distributions of the central members of the order statistics.

The contents of this thesis are presented in three chapters.

Chapter one discusses the necessary and sufficient condition for the convergence of the central members of the order statistics to a limiting distribution function with a fixed sample size. The class of the limiting distributions is obtained and found to be only four types. / Finally, the limiting distributions of the intermediate members of the order statistics are obtained.

The second chapter is a generalization of the first chapter. This chapter deals with the limiting joint distributions of the central and the intermediate and the limiting joint distributions of the central members of the order statistics with a fixed sample size. / It also shows that the limiting joint distributions of the central members are sixteen types, and these central members are asymptotically dependent.

III

This chapter also states an example to show the effect of the difference between ranks on the correlation coefficient. Moreover the intermediate and central members are found to be asymptotically independent. Finally, the class of the limiting joint distributions of the intermediate and central members is obtained.

Chapter three deals with the transfer theorems, the problem in this chapter is different from the problems studied in the first two chapters, where the sample size itself is an integral valued random variable independent of the observations.

When the sample size is a random variable, two transfer theorems are studied. The first transfer theorem of the order statistics deals with the univariate case and the second transfer theorem deals with the bivariate case. These theorems show that from the convergence of the central members of order statistics when the sample size is not random (in the classical case) we get the convergence of the central members of order statistics when the sample size is a random variable (in the non classical case).

An inverse theorem of the first transfer theorem is proved [13]. Two examples for the limiting distribution of the central members of the order statistics when the sample size is a random variable having a geometric distribution are given.

CHAPTER ONE

*THE LIMITING DISTRIBUTIONS OF
THE CENTRAL MEMBERS OF
ORDER STATISTICS*

CHAPTER I

THE LIMITING DISTRIBUTIONS OF THE CENTRAL MEMBERS OF ORDER STATISTICS

This chapter investigates the necessary and sufficient condition for the convergence of the central members of the order statistics to a limiting distribution function with a fixed sample size. Indeed the class of the limiting distributions is obtained and we show that they are only four types. At last the limiting distributions of the intermediate members of the order statistics are studied.

1.1. INTRODUCTION

The limiting distributions of the central members of the order statistics are completely obtained by Smirnov [14] who also studied the class of the limiting distributions of the central members. Tchebycheff [17] studied the class of the limiting distributions of the intermediate members of the order statistics under some specified conditions. Moreover Smirnov [15] determined generally without any assumptions the limiting distributions of the intermediate members of the order statistics.

Let us now present some basic concepts and definitions that are needed in this thesis.

Let $X_1, X_2, \dots, X_n$ be a sequence of independent and identically distributed random variables with cumulative distribution function $F(x) = p(X_k < x)$, $k = 1, 2, 3, \dots, n$. If we rearrange this sequence in an ascending order of magnitude then it follows that,

$$\xi_1^{(n)} \leq \xi_2^{(n)} \leq \xi_3^{(n)} \leq \dots \leq \xi_n^{(n)} \quad (1.1.1)$$

where

$\xi_k^{(n)}$ is the k th order statistics ($k = 1, 2, \dots, n$),

specially

$$\xi_1^{(n)} = \min (X_1, X_2, \dots, X_n),$$

$$\xi_n^{(n)} = \max (X_1, X_2, \dots, X_n).$$

The above sequence is called the order statistics of the observations $X_1, X_2, \dots, X_n$ and it is a non-decreasing sequence.

The rank of the k -th member of order statistics is $\frac{k}{n}$, if $\frac{k}{n} \rightarrow 0$ or 1 , as $n \rightarrow \infty$, then $\xi_k^{(n)}$ and $\xi_{n-k}^{(n)}$ are called the left or the right extreme order statistics respectively, this means that, as $n \rightarrow \infty$, k is constant, $\xi_k^{(n)}$, $\xi_{n-k}^{(n)}$ are called the extreme order statistics.

If $k(n) \longrightarrow \infty$, as $n \longrightarrow \infty$, but $\frac{k(n)}{n} \xrightarrow{n \longrightarrow \infty} \lambda$, $0 < \lambda < 1$ then $\xi_k^{(n)}$ is called the central member of the order statistics, if $k(n) \xrightarrow{n \longrightarrow \infty} \infty$, $\frac{k(n)}{n} \xrightarrow{n \longrightarrow \infty} 0$, then $\xi_k^{(n)}$ is called intermediate member of the order statistics.

Define the cumulative distribution function of the k -th order statistics as follows

$$\begin{aligned} \Phi_{kn}^{(n)}(x) &= \Pr\{X_k \leq x\}, \quad k = 1, 2, 3, \dots, n \\ &= \sum_{m=k}^n \binom{n}{m} [F(x)]^m [1-F(x)]^{n-m} \\ &= \frac{n!}{(k-1)!(n-k)!} \int_0^{F(x)} x^{k-1} (1-x)^{n-k} dx. \quad (1.1.1) \end{aligned}$$

1.2. THE LIMITING DISTRIBUTIONS OF THE CENTRAL MEMBERS OF ORDER STATISTICS IN THE UNIVARIATE CASE.

This section studies the limiting distributions of the central members of the order statistics, it shows that the distribution function of the central members with a suitable choice of normalization with constants converges to some limiting distributions.

In other words this section shows that if

$$\Phi_{kn}^{(n)}(a_n x + b_n) = \Pr(\xi_k^{(n)} < a_n x + b_n) = \Pr\left(\frac{\xi_k^{(n)} - b_n}{a_n} < x\right)$$

for a suitable choice of constants $a_n > 0$, b_n , then $\Phi_{kn}^{(n)}(a_n x + b_n)$ converges to $\Phi(x)$ as $n \rightarrow \infty$, at every continuity points of $\Phi(x)$. To study the above problem exactly, the following theorem must be first proved.

THEOREM 1.2.1

If the rank of $\xi_k^{(n)}$ is $\frac{k}{n}$, $\frac{k}{n} \xrightarrow{n \rightarrow \infty} \lambda$, $0 < \lambda < 1$ and for a suitable choice of constants $a_n > 0$, b_n , the distribution function of the central member $\xi_k^{(n)}$ converges to a limiting distribution function $\Phi(x)$

$$\Phi_{kn}^{(n)}(a_n x + b_n) = \Pr\left(\frac{\xi_k^{(n)} - b_n}{a_n} < x\right) \xrightarrow{n \rightarrow \infty} \Phi(x) \quad (1.2.1)$$

if and only if the following relation is satisfied

$$\tilde{u}_n(x) = \frac{F(a_n x + b_n) - \lambda_{kn}}{\tau_{kn}} \xrightarrow{n \rightarrow \infty} u(x) \quad (1.2.2)$$

where

$$\lambda_k = \frac{k}{n+1}, \quad \nu_{kn} = 1 - \lambda_{kn}, \quad \text{and} \quad \tau_{kn} = \sqrt{\frac{\lambda_{kn} \nu_{kn}}{n+1}},$$