ON THE NONLINEAR ELECTROHYDRODYNAMIC STABILITY OF A CYLINDRICAL SURFACE

A THESIS

Submitted in Partial Pulfilment of the Requirements of the Award of the Master of Science Degree

 $\mathbf{B}\mathbf{y}$

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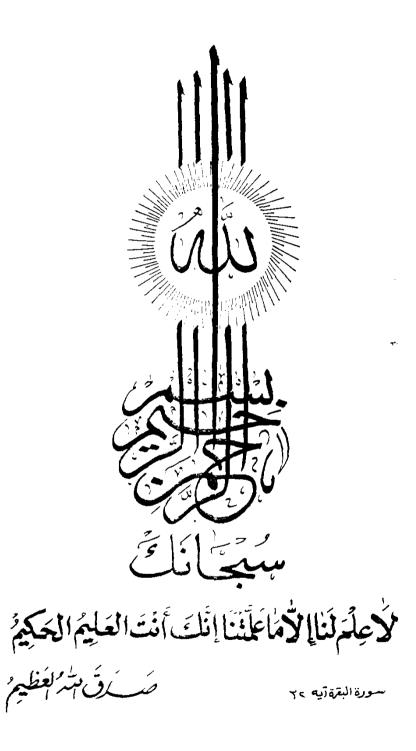
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NOTE

This thesis is submitted to Aim Shams University in partial fulfilment of the requirements of the Master of Science Degree in Applied Mathematics.

Besides the research work in this thesis, the candidate has attended five postgraduate courses within the year (1981 - 1982) including the following topics:

- 1. Quantum Mechanics
- 2. General Relativity
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DEDICATED TO MY PARENTS AND MY WIFE

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SUMMARY

SUMMARY

The thesis is mainly concerned with the nonlinear electrohydrodynamic stability of a system of two dielectric incompressible fluids which have a cylindrical interface when stressed by a constant axial electric field.

This thesis consists of four chapters :

In chapter I, we explain the main aspects of electrohydrodynamics and its various applications. We discuss the concepts of electrohydrodynamics and stability in nonlinear systems. We review but the related perturbation techniques including the method of multiple scale.

Chapter II, examines the nonlinear electrohydrodynamic stability of an interface between two infinite, dielectric and incompressible fluids under the influence of a constant axial electric field. We use the method of multiple scale to expand the various perturbation quantities to yield uniformly valid expansions near the outoff wavenumber.

Chapter III, we reach two Schrödinger equations describing the behaviour of the system. The surface elevation and

the cutoff wavenumber are determined and the stability criterion of the system is also discussed. A comprehensive analysis of the stability of the perturbed system follows using, where possible, a purely mathematical analysis. Where such a procedure has not been found possible, a detailed numerical study is adopted and stability diagrams are drawn for many representative values of the physical parameters.

An important result is that the electric field plays a dual role on the stability conditions for wavenumbers k less than unity. For values of k greater than unity the electric field is destabilizing. The ratio of the dielectric constants affects the stability conditions. It is also observed for the first time for jet problems that the density ratio plays a role on the stability criterion. The appearance of resonance lines in the stability diagram depends on the ratios of the dielectric constants and densities.

In chapter IV, we explain the nonlinear Schrödinger equation obtained from the nonlinear solutions of the cylindrical interface and study the solitary waves and solitons formation through the surface. We also discuss the nonlinear

Schrödinger equation and try to find an approximate solution of the Potential monlinear Schrödinger equation. A new expansion is obtained.

CHAPTER (I) INTRODUCTION

CHAPTER I

INTRODUCTION

1.1. Electrohydrodynamic (EHD) Stability:

Electronypropynamics include that part of fluid mechanics concerned with electrical force effects. It can alternatively be considered as that part of electrodynamics which is concerned with the influence on the moving media. Many of the interesting problems in electrohydrodynamics involve both an effect of the fluid motion on the fields, and conversely, an influence of the fields upon the motion.

A salient feature of electrohydrodynamic interactions is the irrotational nature of the electric field intensity E. Dynamic currents are also so small that the effects of magnetic induction are negligible, (this is the so-called quasi-static approximation).

The equations governing motion of a fluid stressed with electric forces can be written as $[\ 5\].$

$$\rho \left[\frac{\partial v_{i}}{\partial z} + v_{i} - \frac{\partial v_{i}}{\partial x_{i}} \right] = \frac{\partial \left[\frac{v_{i}}{\partial x_{i}} + \rho \right] x_{i}}{\partial \left[\frac{v_{i}}{\partial x_{i}} + \rho \right] x_{i}}, \quad (1.1.1)$$

$$\frac{3s}{3t} + \frac{3}{3t}$$
, $s_{i,j} = 0$. (1.1.2)