ON THE NONLINEAR ELECTROHYDRODYNAMIC STABILITY OF A CYLINDRICAL SURFACE

A THESIS

Submitted in Partial Fulfilment of the Requirements of the Award of the Master of Science Degree

By

MOHAMED IBRAHIM AHMED OTHMAN

Supervisors

Prof.
ABOU EL-MAGD A. MOHAMED
Department of Mathematics
Faculty of Education
Ain Shams University

Prof.
AHMED GALAL EL-SAKKA
Department of Mathematics
Faculty of Science
Ain Shams University

Dr. EL-SAYED F.A. EL-SHEHAWEY
Department of Mathematics
Faculty of Education
Ain Shams University

SUBMITTED TO
AIN SHAMS UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS.

1987

الله

بسم الله الرحمن الرحيم

سبحانك

لا علم لنا إلا ما علمتنا إنك أنت العليم الحكيم

صدق الله العظيم

سورة البقرة آيه ٣٢

بسم الله الرحمن الرحيم

ACKNOWLEDGEMENTS

I am indebted and very grateful to my supervisor professor **ABOU EL MAGD A.** MOHAMED Professor of Applied Mathematics, Faculty of Education, Ain Shams University, for suggesting the problem involved in this work and for his helpful guidance, valuable suggestions and discussions throughout his supervison of this work.

I am deeply grateful to my supervisor professor **AHMED GALLAL EL SAKKA** Professor of Applied Mathematics, Faculty of Science, Ain Shams University, for his keen interest, necessary facilities and encouragement.

I should like to offer deepest gratitude to my co. supervisor Dr. EL SAYED F.A. EL SHEHAWEY Assistant Professor of Applied Mathematics, Faculty of Education, Ain Shams University, for his kind help, helpful guidance and facilities offered throughout this investigation.

NOTE

This thesis is submitted to Ain Shams University in partial fulfilment of the requirements of the Master of Science Degree in Applied Mathematics.

Besides the research work in this thesis, the candidate has attended five postgraduate courses within the year (1981 - 1982) including the following topics :

1. Quantum Mechanics
2. General Relativity
3. Solid state and Hydrodynamics
4. Elasticity
5. Differential Equations.

The applicant MOHAMED IBRAHIM AHMED OTHMAN has successfully passed the final examination of these courses.

SUPERVISORS

Prof. ABOU EL MAGD A. MOHAMED
Department of Mathematics
Faculty of Education
Ain Shams University

Prof. AHMED GALAL EL SAKKA
Department of Mathematics
Faculty of Science
Ain Sham University

Dr. EL SAYED F.A.EL SHEHAWEY
Department of Mathematics
Faculty of Education
Ain Shams University

VITAE

Mohamed Ibrahim Ahmed Othman was born in September 5, 1958, in Mit Ghamr Dakahlia, Egypt.

He attended Mit Ghamr Secondary School from 1973 till 1976, and got the Secondary School Certifcate in 1976.

He was enrolled in the Faculty of Science, Zagazig University, Egypt, where he majored in Mathematics and received the Bachelor of Science in Mathematics in 1980.

In October, 1980, he was appointed as a demonstrator in Mathematics Department, Faculty of Science, Zagazig University.

He was registered for the degree of M.Sc. in Applied Mathematics in July 18, 1983 under the supervision of Prof. Abou El Magd A.M., Professor of Applied Mathematics, Faculty of Education, Ain Shams University and Prof. Ahemd Galal El Sakka, Professor of Applied Mathematics Faculty of Science Ain Shams University and Dr. El Sayed F.A. El Shehawey Assistant Professor of Applied Mathematics Faculty of Education Ain Shams University.

DEDICATED TO

MY PARENTS AND MY WIFE

CONTENTS

CHAPTER (III)

SURFACE BEHAVIOUR AND STABILITY ANALYSIS

CHAPTER (IV)

CYLINDRICAL ELECTROHYDRODYNAMIC SOLITARY WAVES

SUMMARY

SUMMARY

The thesis is mainly concerned with the nonlinear electrohydrodynamic stability of a system of two dielectric incompressible fluids which have a cylindrical interface when stressed by a constant axial electric field.

This thesis consists of four chapters :

In chapter I, we explain the main aspects of electro-hydrodynamics and its various applications. We discuss the concepts of electrohydrodynamics and stability in nonlinear systems. We review out the related perturbation techniques including the method of multiple scale.

Chapter II, examines the nonlinear electrohydrodynamic stability of an interface between two infinite, dielectric and incompressible fluids under the influence of a constant axial electric field. We use the method of multiple scale to expand the various perturbation quantities to yield uniformly valid expansions near the cutoff wavenumber.

Chapter III, we reach two Schrödinger equations describing the behaviour of the system. The surface elevation and

the cutoff wavenumber are determined and the stability criterion of the system is also discussed. A comprehensive analysis of the stability of the perturbed system follows using, where possible, a purely mathematical analysis. Where such a procedure has not been found possible, a detailed numerical study is adopted and stability diagrams are drawn for many representative values of the physical parameters.

An important result is that the electric field plays a dual role on the stability conditions for wavenumbers k less than unity. For values of k greater than unity the electric field is destabilizing. The ratio of the dielectric constants affects the stability conditions. It is also observed for the first time for jet problems that the density ratio plays a role on the stability criterion. The appearance of resonance lines in the stability diagram depends on the ratios of the dielectric constants and densities.

In chapter IV, we explain the nonlinear Schrödinger equation obtained from the nonlinear solutions of the cylindrical interface and study the solitary waves and solitons formation through the surface. We also discuss the nonlinear

Schrödinger equation and try to find an approximate solution of the Potential nonlinear Schrödinger equation. A new expansion is obtained.

CHAPTER (I)

INTRODUCTION

CHAPTER I

INTRODUCTION

1.1. Electrohydrodynamic (EHD) Stability :

Electrohydrodynamics include that part of fluid mechanics concerned with electrical force effects. It can alternatively be considered as that part of electrodynamics which is concerned with the influence on the moving media. Many of the interesting problems in electrohydrodynamics involve both an effect of the fluid motion on the fields, and conversely, an influence of the fields upon the motion.

A salient feature of electrohydrodynamic interactions is the irrotational nature of the electric field intensity $\vec{E}$. Dynamic currents are also so small that the effects of magnetic induction are negligible, (this is the so-called quasi-static approximation).

The equations governing motion of a fluid stressed with electric forces can be written as [5].

$$\rho \left[\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right] = \frac{\partial T_{ij}}{\partial x_j} + \rho X_i , \qquad (1.1.1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho v_j) = 0 . \qquad (1.1.2)$$