ON LINE GRAPHS

THESIS

Submitted in Partial Fulfilment for the Degree

of

Master of Science in Mathematics

By

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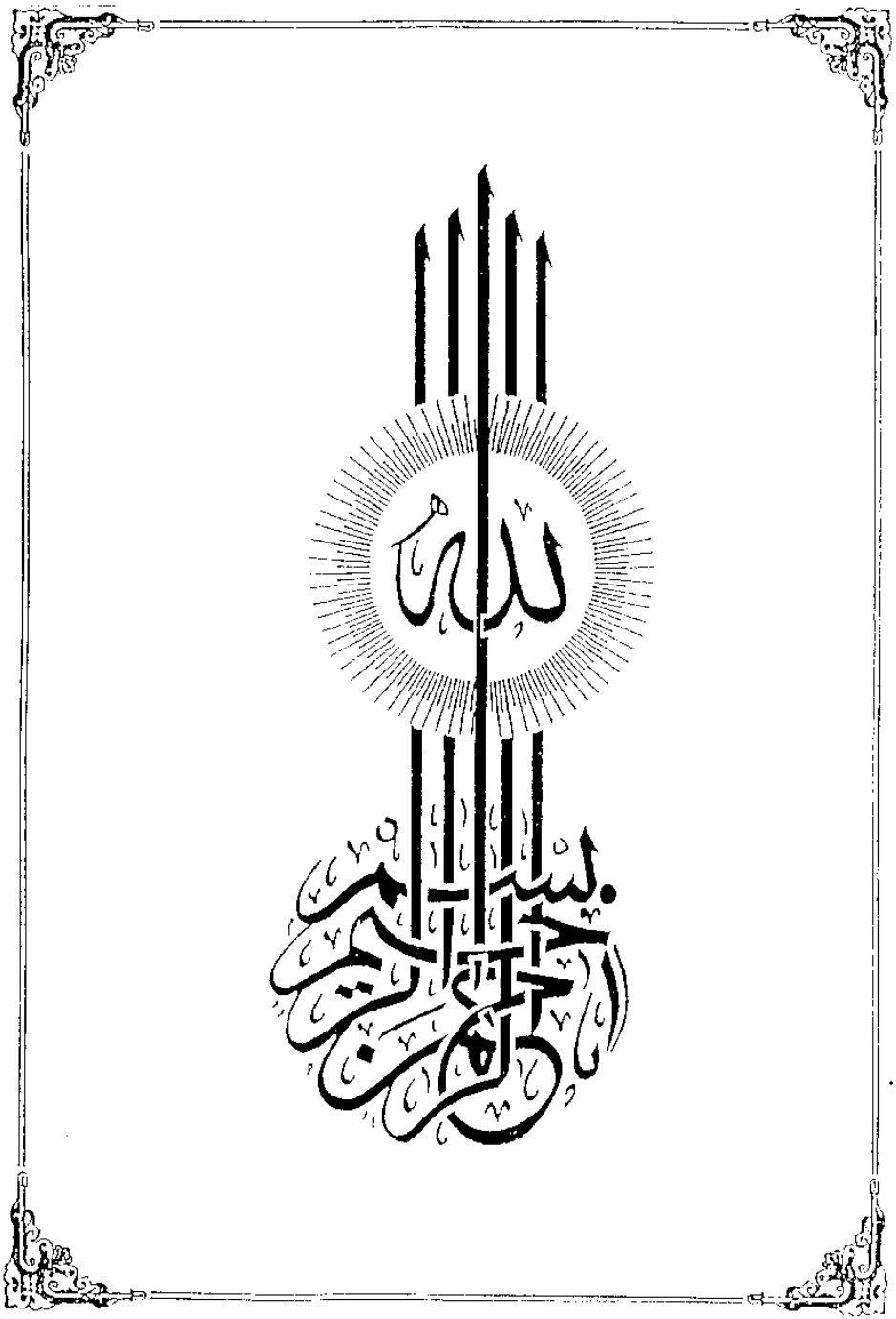

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ON LINE GRAPHS

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SUMMARY

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SUMMARY

In entirely new theory, that of the diagrammatic "Graph" can be of great assistance in dealing with those combinatorial problems which occur in various economic, sociological, or technological fields. Indeed, the realization of the value of this "theory of Graphs" has given it a place of great importance in education. It is, perhaps ,that aspect of the theory of sets which can produce the most fruitful results, not only for the "pure mathematician ", but also for some other fields such as Engineering, organization, biology, psychology, sociology, geograply, theoretical physics, nuclear physics, biomathematics, linguistics, zoology, computer science, anthropology and many others.

The recent years have witnessed a remarkable growth in the applications of graph theortical principles to chemistry. There are several reasons for the increasing popularity of graph theory in chemistry. First, there is hardly any concept in the natural sciences which is closer to the motion of graph than the structural (constitutional) formula of a chemical compound, because a graph is simply said, a mathematical structure which

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may be used directly to represent a molecule when the only property considered is the internal connectivity, i.e. whether or not a chemical bond joins two atoms in a molecule. Here, the chemical bond is represented by only a line connecting two atoms. Second, graph theory provides simple rules by which experimental chemists may obtain many useful qualitative predictions about the structure and reactivity of various compounds. Third, graph theory may be used as a foundation for the representation and categorization of a very large number of chemical systems. However, the language of graph theory is different from that of chemistry,we offer a short glossary in Table 2.4.1. Chapter 2, which contains the terminology of graph theory which we propose for standard use in chemistry and the corresponding chemical terms.

Here we discuss mainly "line Graphs" and something about "Isometric Graphs".

This thesis contains four chapters. The first chapter presents the main terms and concepts of graph theory, such as : graphs, complete graphs, bipartite graphs, circuit graph, isomorphism of the graphs,paths

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circuits and valencies in graphs, subgraphs, trees, planarity, outerplanar and maximal outerplanar graphs, thickness, duality Eulerian and Hamiltonian graphs, subdivision graph homeomorphic graphs, crossing number, square of a graph, square root of a graph, clique graph, girth, the composition operation and regularity of graph.

In the second chapter some applications are presented such as the minimal connector problem and the traveling salesman problem. The minimal articulated set is discussed.

Some chemical applications, namely:the correspondence between the graph theoretical and chemical terms, molecular graphs, molecular topology, molecular orbitals, computation of characteristic polynomials of graphs, Caterpiller tree, the Clar graph , line graph and sexted polynomial are considered.

The third chapter deals with some basic results on line graphs, such as the characterization of line graphs, forbidden subgraphs in planar repeated line graphs, Hamiltonian index and the Hamiltonian line graphs. Graph isomorphic to subgraphs of their line

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graphs, A 1-factorization of the line graphs of complete graphs and the 1-factorization of some line graphs, connected of graphs switching equivalent to their itereated line graphs, and the forbidden naked sub-graphs are considered.

In the fourth chapter we studied some remarks and lemmas concerning line graphs and isometric graphs. An inequality for the thickness of a line graph is given. Also two simple line graph equations are discussed. This chapter is published in "The Journal of the Faculty of Education Ain Shams University, No.9, pp. 177-186, 1985".

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CHAPTER I

ELEMENTS OF GRAPH THEORY

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This chapter givgs the basic definitions, theorems, and conceptes of graph theory.

1.1. Definitions

(i) A simple graph G is defined to be a pair (V (G),E(G)) where V (G) is a non-empty finite set of elements called vertices(nodes,or points), and E (G) is a finite set of unordered pairs of distinct elements of V (G) called edges (or lines); V (G) is sometimes called the vertex-set and E (G) the edge-set of G. For example, Fig. (1.1) represents the simple graph G whose vertex -set is the set

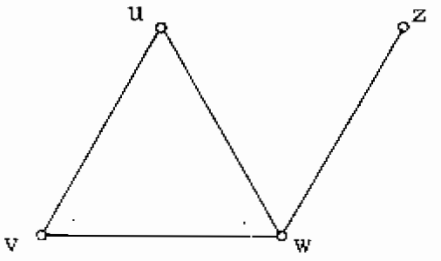

Fig. (1.1)

$\{u,v,w,z\}$ and whose edge set E (G) consists of the pairs $\{u, v\}, \{v, w\}, \{u, w\}$ and $\{w, z\}$.

(ii) A graph whose edge-set is empty is called a null graph as shown in Fig. (1.2).

Fig. 1.2

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