"Treatment of Delay - Differential Equations And Its Accuracy"

THESIS

Submitted For The Degree

Of

DOCTOR OF PHILOSOPHY (Ph. D.)

Pure Mathematics

By

Wafaa Mostafa Mohamed Ahmed

Mathematics Department, College Of Women

Ain Shams University

Supervised by

Professor Dr. Hassan Nasr

Professor of Mathematics

Benha Higher Institute of Technology

Professor Dr. F. A. Ayoub

Professor of Mathematics

Ain Shams University

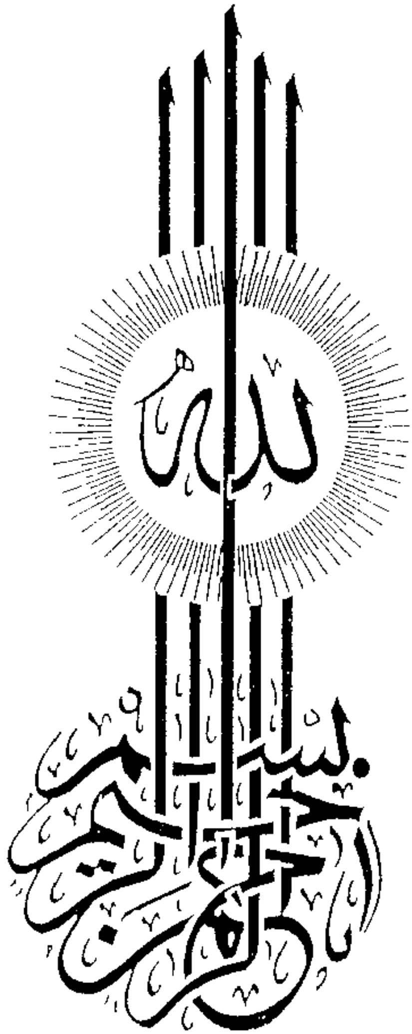
Dr. Mohamed Kouta

Ass. Prof. for Computer Scince

Mulitary Technical College

1993

TO THE MEMORY OF MY FATHER AND
TEACHER PROF. DR. A. I. ABD EL-KARIM


بسم الله الرحمن الرحيم

Acknowledgements

My gratitude, first of all, is due to Allah for giving me inspiration and ability to introduce this work.

I wish to express my deepest appreciation and sincere gratitude to Prof. Dr. Hassan Nasr Professor of Mathematics Benha Higher Institute of Technology not only for his suggesting and super-vising this work, but also for his excellent helps, his keen interest and continuous encouragement.

I also have the pleasure for thanking Prof. Dr. Farouk Ayoub Head of Mathematics Department, College of Women, Ain Shams University and I wish him all the best for his aggoreciated and grateful help he provided me.

I wish to express my deep thanks and gratitude to Dr. Mohamed Kouta, Department of Computer Military Technical College and Dr. Gamal Ali and all staff members of mathematics department in faculty of Women, Ain Shams University.

CONTENTS

Summary

This thesis deals with the treatment of delay differential equations and its accuracy in six chapters.

The first chapter is considered as an introduction and it contains the fundamentals for delay-differential equations. We have studied the fields of application, some problems from practice and test problems for checking the used numerical method and at the and of the chapter we have studied the general form of the differential difference equation.

The second chapter deals with the existence and uniqueness theorems:We consider the delay-differential equation

$$a_0 y(t) + b_0 y(t) + b_1 y(t-r) = f(t) , \qquad t \geq r$$

with the initial function

$$y(t) = g(t) , \qquad \forall\ t \in [0,r] ,$$

and we investigate the existence and uniqueness of solutions,the condition of continuity of the derivative of the solution at r , the conditions of continuity of the second derivative of the solution at 2r, the domain of the solution of delay differential equations. We also study the polynomial solutions and the exponential solutions. At the end of the chapter we study the transformation of the general delayed differential equations to a simpler one.

In the third chapter we have studied the linear systems of delay-differential equations. In this chapter we could transform a higher order delay-differential equation to a system of first order, and we have studide the classification of the linear systems of differential-differeance equations, existence and uniqueness of solutions for the system of delay differential equations and the characteristic matrix function of this system. Finally a priori estimate of the norm of the solution of a system of delay-differential equations is derived.

In the fourth chapter we have studied the modified methods for the numerical treatment of delay-differential equations. In this chapter we could derive the Predictor - Modifier-Corrector-Final Value method for solving the delay differential equation

$$y'(t) = f(t\ ,\ y(t)\ ,\ y(t-r))\ , \qquad t \geq r$$

$$y(t) = g(t)\ , \qquad \forall\ t \in [0,r].$$

and for the systems of delay differential equations

$$\underline{Y}'(t) = \underline{F}(t, \underline{Y}(t), \underline{Y}(t-r))\ , \qquad t \geq r$$

$$\underline{Y}(t) = \underline{G}(t) \qquad \forall\ t \in [0,r],$$

we have also studied special cases of the Predictor-Modifier-Corrector-Final Value method and have solved examples on a delay-differential equation and a system of delay-differential equations.

In chapter five we have studied the stability of the numerical methods used for the delay-differential equations. The study includes the asymptotic properties of the solution of linear delay-differential equations and the stability conditions of the numerical methods used/ The numerical method used involves the construction of general multistep methods of special properties.

In chapter six we have studied the stability regions of the methods of the delay equations. We have studied first the stability region for second and third order predictor-corrector methods and then the stability region for third and fourth order modified predictor-corrector methods.

the Micro VAX comuter (VT. 3100) was used for computing the numerical results.

Part of this work is already published in [30] the proceedings of the 17th international conference of computer science and statistics, Vol. 6, pp. 209-218, Cairo 1992. In addition two further papers [31] and [32] are published in the proceedings of the 18th international conference of computer science and statistics, Cairo, (1993).

We hope to continue the work by further investigations for solving the delay differential equations by higher order modified-predictor-corrector-final value methods to be treated in future.

CHAPTER I

INTRODUCTION AND FUNDAMENTALS

1.1 Fields of Application of Delay - Differential Equations

The Fundamental problem to analyze a retarded process from the real world, to give a description by a mathematical model and to determine the subsequent (future) behavior leads to delay differential equations. Detailed studies of the real world force us , to take account of the fact that the rate of change of physical systems depend not only on their present state , but also on their past history.The research in the very last years shows that various approximation and also optimization problems appear in the numerical treatment of delay equations. Such type of equations appear in many fields of applications:

In **Physics And Engineering** the applications are in the : nuclear reactions, electrostatic charge problems, elasticity theory, automatic controls, diffusion, rocket engines and transistor design.

In **Biology** the applications are in the : population growth, ecological models, host - parasite -models and two-species competition.

In **Medicine** the applications are in the : pharmacokinetic models, spread of infection diseases andproduction of red blood cells.

In **Economics** the applications are in the : business cycles and economic growth.

Analytic Number Theory