

"Treatment of Delay - Differential Equations And Its Accuracy"

THESIS

Submitted For The Degree
Of
DOCTOR OF PHILOSOPHY (Ph. D.)
Pure Mathematics

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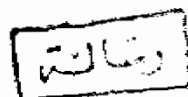
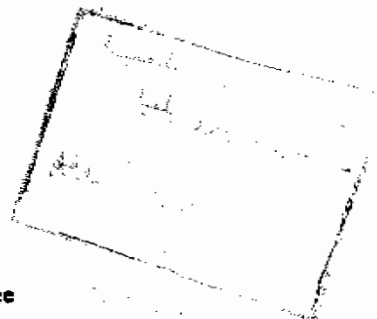
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TO THE MEMORY OF MY FATHER AND
TEACHER PROF. DR. A. I. ABD EL-KARIM



Acknowledgements

My gratitude, first of all, is due to Allah for giving me inspiration and ability to introduce this work.

I wish to express my deepest appreciation and sincere gratitude to Prof. Dr. Hassan Nasr Professor of Mathematics Benha Higher Institute of Technology not only for his suggesting and super-vising this work, but also for his excellent helps, his keen interest and continuous encouragement.

I also have the pleasure for thanking Prof. Dr. Farouk Ayoub Head of Mathematics Department, College of Women, Ain Shams University and I wish him all the best for his aggoreciated and grateful help he provided me.

I wish to express my deep thanks and gratitude to Dr. Mohamed Kouta, Department of Computer Military Technical College and Dr. Gamal Ali and all staff members of mathematics department in faculty of Women, Ain Shams University.

CONTENTS

	Page
SUMMARY.....	vi
 CHAPTER I : Introduction And Fundamentals 	
1.1 Fields of application of delay differential equations	1
1.2 Some problems from practice.....	2
1.2.1 Population growth.....	2
1.2.2 Control systems.....	3
1.3 Test problems for checking the applied numerical method.....	5
1.4 General form of differential difference equations....	8
1.5 Equations of delayed , neutral and advanced type.....	10
1.6 Basic definitions and conventions.....	11
 CHAPTER II : Existence and Uniqueness Theorems 	
2.1 The considered delay-differential equation.....	13
2.2 The Existence and uniqueness of solutions	13
2.3 Existence and uniqueness of solutions for a finite set of simple discontinuities.....	16

2.4	The condition of the derivative of the solution at r	17
2.5	Conditions of continuity of the second derivative of the solution at $2r$	17
2.6	Simplifying the work.....	18
2.7	The domain of the solution of delay differential equations.....	19
2.8	General study of polynomial solutions.....	21
2.9	Exponential solutions and Delayed Operator.....	23
2.10	Transformation of the general delay differential equations to a simpler one.....	25

CHAPTER III : Linear Systems of Delay-Differential equations

3.1	Transforming a higher order delay-differential equation to a system of first order.....	28
3.2	Classification of linear systems of differential difference equations.....	32
3.3	Existence and uniqueness of solutions for the system of delay equations.....	33
3.4	The characteristic matrix function $H(S)$ of a system of delay-differential equations.....	34

3.4.1	The derivatives of the characteristic matrix function $H^{(k)}(S)$	36
3.4.2	The effect of the operator L on $t^n e^{st} C$	37
3.4.3	The effect of the operator L on $\sum_{k=0}^n t^{n-k} e^{st} C$	38
3.5	A priori estimate of the norm of the solution of systems of delay-differential equations	40

CHAPTER IV :Modified Methods for Numerical Treatment of Delay Differential Equations

4.1	Introduction.....	48
4.2	Derivation of The Predictor-Modifier-Corrector-Final Value Method for Solving a Delay Differential Equation	49
4.2.1	Derivation of the Method.....	49
4.2.2	The Order of Accuracy of the Method.....	53
4.3	Numerical Example.....	55
4.4	General Predictor-Modifier-Corrector Final Value Method for Solving Systems of Delay Differential Equations.....	58
4.5	Special Examples of Predictor-Modifier-Corrector-Final Value Methods.....	61
4.5.1	Third Order Two Step Modified Predictor-Corrector Method.....	61

4.5.2 Fourth Order Two Step Modified Predictor-Corrector Method.....	63
4.6 Numerical Example.....	65

CHAPTER V :The stability of The Method of The Delay Equations

5.1 Introduction.....	69
5.2 The asymptotic properties of the solution of a linear delay - differential equation.....	69
5.3 The stability conditions for the delay differential equations.....	73
5.4 Stability analysis of the numerical methods for solving a delay differential equation.....	75
5.5 The Construction of general multistep methods of special properties.....	81
5.5.1 Second order-two step method.....	83
5.5.2 Third order-two step method.....	86

CHAPTER VI : Stability Region for Specific Predictor-Corrector Methods for Solving Delay Differential Equations

6.1 Introduction.....	90
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6.2	Stability region for a second order predictor-corrector method.....	90
6.2.1	The single delay.....	91
6.2.2	The double delay.....	96
6.3	Stability region for a third order predictor-corrector method.....	97
6.3.1	The single delay.....	98
6.3.2	The double delay.....	100
6.4	Stability region for a third order modified predictor-corrector method for solving Delay Differential Equations.....	102
6.4.1	The single delay.....	106
6.4.2	The double delay.....	107
6.5	Stability region for a fourth order modified predictor-corrector method for solving Delay Differential Equations.....	107
6.5.1	The single delay.....	113
6.5.2	The double delay.....	113

Summary

This thesis deals with the treatment of delay differential equations and its accuracy in six chapters.

The first chapter is considered as an introduction and it contains the fundamentals for delay-differential equations. We have studied the fields of application, some problems from practice and test problems for checking the used numerical method and at the end of the chapter we have studied the general form of the differential difference equation.

The second chapter deals with the existence and uniqueness theorems: We consider the delay-differential equation

$$a_0 y(t) + b_0 y(t) + b_1 y(t-r) = f(t), \quad t \geq r$$

with the initial function

$$y(t) = g(t), \quad \forall t \in [0, r],$$

and we investigate the existence and uniqueness of solutions, the condition of continuity of the derivative of the solution at r , the conditions of continuity of the second derivative of the solution at $2r$, the domain of the solution of delay differential equations. We also study the polynomial solutions and the exponential solutions. At the end of the chapter we study the transformation of the general delayed differential equations to a simpler one.

In the third chapter we have studied the linear systems of delay-differential equations. In this chapter we could transform a higher order delay-differential equation to a system of first order, and we have studied the classification of the linear systems of differential-difference equations, existence and uniqueness of solutions for the system of delay differential equations and the characteristic matrix function of this system. Finally a priori estimate of the norm of the solution of a system of delay-differential equations is derived.

In the fourth chapter we have studied the modified methods for the numerical treatment of delay-differential equations. In this chapter we could derive the Predictor - Modifier-Corrector-Final Value method for solving the delay differential equation

$$y'(t) = f(t, y(t), y(t-r)), \quad t \geq r$$

$$y(t) = g(t), \quad \forall t \in [0, r].$$

and for the systems of delay differential equations

$$\underline{Y}'(t) = \underline{F}(t, \underline{Y}(t), \underline{Y}(t-r)), \quad t \geq r$$

$$\underline{Y}(t) = \underline{G}(t) \quad \forall t \in [0, r],$$

we have also studied special cases of the Predictor-Modifier-Corrector-Final Value method and have solved examples on a delay-differential equation and a system of delay-differential equations.

In chapter five we have studied the stability of the numerical methods used for the delay-differential equations. The study includes the asymptotic properties of the solution of linear delay-differential equations and the stability conditions of the numerical methods used. The numerical method used involves the construction of general multistep methods of special properties.

In chapter six we have studied the stability regions of the methods of the delay equations. We have studied first the stability region for second and third order predictor-corrector methods and then the stability region for third and fourth order modified predictor-corrector methods.

the Micro VAX computer (VT. 3100) was used for computing the numerical results.

Part of this work is already published in [30] the proceedings of the 17th international conference of computer science and statistics, Vol. 6, pp. 209-218, Cairo 1992. In addition two further papers [31] and [32] are published in the proceedings of the 18th international conference of computer science and statistics, Cairo, (1993).

We hope to continue the work by further investigations for solving the delay differential equations by higher order modified-predictor-corrector-final value methods to be treated in future.

CHAPTER I

INTRODUCTION AND FUNDAMENTALS

1.1 Fields of Application of Delay - Differential Equations

The Fundamental problem to analyze a retarded process from the real world, to give a description by a mathematical model and to determine the subsequent (future) behavior leads to delay differential equations. Detailed studies of the real world force us , to take account of the fact that the rate of change of physical systems depend not only on their present state , but also on their past history. The research in the very last years shows that various approximation and also optimization problems appear in the numerical treatment of delay equations. Such type of equations appear in many fields of applications:

In Physics And Engineering the applications are in the : nuclear reactions, electrostatic charge problems, elasticity theory, automatic controls, diffusion, rocket engines and transistor design.

In Biology the applications are in the : population growth, ecological models, host - parasite -models and two-species competition.

In Medicine the applications are in the : pharmacokinetic models, spread of infection diseases and production of red blood cells.

In Economics the applications are in the : business cycles and economic growth.

Analytic Number Theory